The Illinois Mathematics Teacher

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The Illinois Mathematics Teacher is the official journal of the Illinois Council of Teachers of Mathematics and is devoted to providing ideas and information to support the professional development of teachers at all levels of the curriculum (K-16). Current issues and select back-issues are available online at http://ictm.org/imt.html.

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Editors’ Note

Welcome to volume 62 of Illinois Mathematics Teacher, the journal of the Illinois Council of Teachers of Mathematics. This marks our first full issue as editors of the journal, and we are excited to continue the tradition of quality scholarship that this journal represents.

During our transition into the editorship, we received invaluable help from many sources from across ICTM. In particular, we would like to thank Marilyn Hasty and Tammy Voepel, who served for a long time as editors of the journal and provided us with valuable resources and insight. We would also like to mention Don Porzio, who provided us with administrative guidance as former president of the ICTM, Ann Hanson, who recommended us for the task and continues to support us in many ways, and Don Beaty, who manages the online platform for the journal.

In this issue

Each year at the annual meeting of the ICTM, winners of the ICTM awards are invited to present remarks to the audience in attendance. Among the awardees this year was Susan Brown, who submitted her inspirational remarks to the journal for publication. Andrzej Sokolowski presents an inquiry-based lesson for exploring linear geometry in “Constructivist Approach to Algebraically Expressing a Line.” Linda Fosnaugh and Patrick Mitchell argue for grounding mathematical procedures in conceptual understanding in “Teaching Tricks Can Be Tricky.” In “Cracking the Codes of Number Relationships,” Lara Willox and Sarah C. Newman present the results of their work developing number sense in early mathematics classrooms. In “iPad Fun: Exploring Slope and Functions Using iPads with the TI-Nspire App,” Ann Wheeler and Brandi Falley present a classroom activity in which students use iPad software to analyze the slopes of real objects. The final contribution to this issue is a mathematics-themed crossword puzzle composed by Christopher Shaw.

Finally, as of this issue, the journal will be published exclusively online through the ICTM website. New articles will be published as they are accepted on the journal website and periodically collected into complete issues. With this transition, several other functions have also moved online. If you would like to join ICTM, we encourage you to visit the ICTM website at http://ictm.org. To submit an article or volunteer to be a reviewer for the IMT, please go to the journal website at http://www.ictm.org/journal where you will find a detailed set of guidelines for submission to the journal. You may also send an email to the editors at imt@ictm.org.

Thank you for reading, and we look forward to working with you.

Daniel Jordan & Christopher Shaw
From the editors: The editors congratulate the winners of the 2014 ICTM awards, which were presented at the 64th Annual Meeting of the Illinois Council of Teachers of Mathematics. The awardees are listed below. We also present the remarks delivered by Susan Brown, the winner of the 2014 T.E. Rine Secondary Mathematics Teaching Award, who submitted the text of her acceptance speech to the IMT. Information about the awards and nomination instructions can be found at [http://ictm.org/ictmawards/](http://ictm.org/ictmawards/).

Max Beberman Mathematics Educator Award
Alan Zollman  
Northern Illinois University, DeKalb

Fred Flener Award
Paul J. Karafiol  
Walter Payton College Prep High School, Chicago

T.E. Rine Secondary Mathematics Teaching Award
Susan Brown  
York High School, Elmhurst

Distinguished Life Achievement in Mathematics
Angela Andrews  
National Louis University - Retired

Illinois Promising New Teacher of Mathematics
Esther Song  
Niles West High School, Niles

Lee Yunker Mathematics Leadership Award
David Spangler  
McGraw-Hill Education, Chicago

Elementary Mathematics Teaching Award
Denise Brown  
Carruthers Elementary School, Murphysboro

Middle School Mathematics Teaching Award
Annie Forest  
Freedom Middle School, Berwyn

A Thought About Teaching (for Math Teachers)  
Remarks by Susan Brown

The first time I taught geometry was many years ago. The textbook was very theoretical, a holdover from the New Math era. Every time we encountered a new relationship, students were asked to determine whether it was an equivalence relationship, and if it was, they had to prove it. I had never heard the term before, and had to learn that an equivalence relationship is one that is reflexive, symmetric, and transitive. For instance, early in the year we studied congruence, and we proved the following:

Reflexive Property:  
\[ \angle A \simeq \angle A. \]

Symmetric Property:  
If \( \angle A \simeq \angle B \), then \( \angle B \simeq \angle A \).

Transitive Property:  
If \( \angle A \simeq \angle B \) and \( \angle B \simeq \angle C \), then \( \angle A \simeq \angle C \).

My students did not find this to be very interesting, and I admit that I did not either. (The only intriguing situation was perpendicularity. Unlike congruence and similarity, it is not an equivalence relationship.) But years afterward, this experience led me to a thought that I did find interesting: Teaching is an equivalence relationship. Let me convince you that this is true.

Reflexive Property: Person A teaches person A. As beginning teachers quickly find out, you must teach yourself an enormous body of knowledge in order to do your job. You must enlarge what you know about mathematics as you struggle to answer questions like “Why does it work this way?” You must formulate alternate explanations to things that you took for granted as a student,
or learned on the surface and never really pondered. Beyond the subject matter, much of what you must teach yourself is about learning and about kids. How do you do these things? Of course, collaboration, professional reading, and further coursework play a role. But they have limited impact unless you also watch yourself and what unfolds in your classroom, and think deeply about what you observe.

Symmetric Property: If person A teaches person B, then person B teaches person A. Perhaps the best teachers you have are your own students. If I have any advice for new teachers, it is to always, always, always listen to your students. Their thinking is the end product of what you do. Their words and actions are the clues that let you discover what they know. Their questions and comments, however odd and puzzling, help you to understand what’s really going on. And they can have wonderful insights and ideas that you never even considered.

Transitive Property: If person A teaches person B, and person B teaches person C, then person A teaches person C. I can still hear the voice of my own geometry teacher in what I say to students. I hope that I have been able to capture some of her enthusiasm and passion for mathematics, and to pass them on to the students and teachers with whom I work. But that is not all. Through my teacher there is a chain that leads backward to Euclid and his predecessors. That chain also moves forward. My students will someday play a role in the early mathematical education of their own children. Some of my students have gone on to become math teachers themselves, and will influence thousands of students in the future. What we do as teachers does not end with us.

Q.E.D.

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Illinois Mathematics Teacher
Constructivist Approach to Algebraically Expressing a Line

Andrzej Sokolowski

Abstract
Although sketching and building linear functions surfaces in many mathematics classes, taking direct measurements leading to function formulation is not often exercised. This paper—written in the form of a lesson accompanied by commentaries—is to fill in the gap. Guided by constructivist learning theory, the lesson places the students in the roles of scientists who integrate their mathematical theoretical knowledge with their skills of measuring to produce algebraic representations of straight lines. The unit does not involve complex equipment, except rulers that are readily available in any math classroom. Despite its simplicity, this lesson generates an engaging, discovery type learning environment that involves not only the students but also the teacher.

Keywords: mathematics, education, constructivism, slope

1. Introduction
Several studies (e.g., Stanton & Moore-Russo, 2012; Lobato & Siebert, 2002) have revealed that the concept of slope, despite its multiple applications, appears to students as an abstract mathematical entity. It is hypothesized that this difficulty might be rooted in presenting the slope as a ratio of dimensionless quantities, which diminishes its physical interpretation and weakens its relevance to students’ prior experiences. This paper presents a lesson that is designed to enrich the meaning of slope—and consequently the meaning of linear functions—by applying direct measurement.

The lesson is rooted in the constructivist theory of knowledge acquisition. Constructivists claim that learners construct their own knowledge based on received impulses (Von Glasersfeld, 1995). One of the opportunities to apply this learning theory is to place learners in realistic settings that resonate with their prior experiences and let them construct the knowledge.

The lesson presented here will reflect the constructivist approach. It can serve as a unit summarizing the process of building linear functions. It can be conducted in any math class provided students are familiar with the concepts of slope and the algebraic formulation of linear functions using slope-point or slope-intercept form.

2. Lesson Description
2.1. Lesson Outline
The commentary provided in the body of the manuscript is based on a lesson conducted with a group of precalculus students in a rural East Texas high school. During the lesson, the students—guided by the teacher—unfolded the physical meanings of all parameters and processes needed to express a given line in a symbolic algebraic form. A metric ruler was used to quantify the needed parameters, such as slope and intercepts. Measurement, which is an element of scientific inquiry, has received significant attention in the newly developed Common Core State Standards (Porter et al, 2011). Thus, including this element in the lesson enhanced not only the constructivist approach but also reflected new mathematics teaching recommendations. In addition, the students used graphing calculators to verify the derived mathematical models.

Since measuring is intertwined with the algebraic meanings of these processes in this lesson, it was hypothesized that a unified
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view of expressing a physical object—a line—algebraically would emerge in students’ minds as a result. It was also expected that, during this process, the mathematical tools used to express the slope of a line in mathematical form would become tangible and more meaningful to students.

The lesson made use of guided discovery as the method of instruction. Thus, the teacher was available to guide the students through the process of transferring a sketched line into its algebraic form.

2.2. Using Measurement to Connect with Students’ Prior Experiences

With the blackboard cleaned of any coordinate axes, the teacher opened the lesson by drawing a line on the board and positing the following question: how can one construct an algebraic equation of the line? The students were puzzled at first because no axes or grid lines were provided—i.e., they experienced a dissonance that prompted a discussion. The teacher directed students’ attention to a general structure of a linear function, $y - y_1 = m(x - x_1)$, and asked how to find the parameters that are needed to use the structure. Students suggested that selecting any two points from the line was necessary. The teacher selected two points and labeled them on the line (see figure 1) and then asked for the next step needed to find the line’s equation. The students were again in doubt because, even though the points were labeled, the coordinates were still not quantified, i.e., the line was placed in an environment without associated mathematical representations that the students were used to having provided for them.

The teacher asked what was needed to establish these points’ numerical coordinates. The students realized that a frame of reference called the “$x$-$y$ coordinate axes” was missing. The teacher then asked where exactly the axes should be drawn. The students suggested locating them near the line. The teacher drew, or asked a student to draw, the coordinate axes on the board in a standard vertical and horizontal fashion, as depicted in figure 2.
Since neither the scaling of the axes nor grid lines were provided, the students experienced yet another dissonance resulting in their inability to continue with the process of the line quantification. Some students suggested “making up a scale” on the $x$ and $y$ axes. The teacher asked if there was a more precise method of finding exact numerical magnitudes of the coordinates. The students had difficulty associating the coordinates with their measurable physical lengths. In order to make the transition, the teacher referred to a meter-stick and said that since the line was positioned with reference to the established $x$-$y$ axes, certain quantities such as lengths could be measured in order to quantify the inclination known as slope and the line’s exact location with reference to the axes. This was a pivotal element of the lesson and apparently “an eye opener” for some students. They realized that the commonly given coordinates represent, in fact, physical distances of points from an established reference, called the origin in mathematics.

A short discussion of what measuring system to use—initiated by the teacher—also took place. Students realized that whether they used centimeters or inches, the equation of the line, although expressed in different units, should resemble the same position and inclination as the line sketched on the board.

In order to fully experience this idea, the students were further guided through the process of deciding what segments to measure and how to measure the segments (see figures 3 and 4). The teacher asked some students to participate and measure the necessary lengths.

The students decided to quantify the vertical intercept first. Its magnitude as measured from the origin was 20 cm. They concluded that the next step was obtaining a numerical value for the slope, which was possible in two ways—either by measuring the coordinates of the points or by measuring the lengths of the run and rise between the points. Most of the students suggested measuring the run and rise between the points. The teacher sparked conversation by asking if measuring the length of the segment $AB$ would be significant in the process of slope quantification. Most students rejected this idea. Yet, as a right triangle was “formed,” this question generated a further inquiry about the differences between the concepts of slope and hypotenuse. Not all of the students understood that these two fundamental concepts were different. After a short discussion consensus was reached that the segment $AB$, when representing the slope, was a ratio, while, as a hypotenuse of a triangle, it represented a length.

2.3. Integrating the Measurements into Symbolic Line Representation

The inclination of the line was determined by the ratio of the vertical and horizontal distances between the points, i.e.,

$$\text{slope} = \frac{\text{vertical displacement}}{\text{horizontal displacement}} = \frac{\Delta y}{\Delta x}.$$
In the example, the slope took the value of \( \frac{43 \text{ cm}}{50 \text{ cm}} = 0.86 \). By applying the slope-intercept form, the students concluded that the equation of the line was \( y = 0.86 \frac{\text{cm}}{\text{cm}} x + 20 \text{ cm} \), or more simply, \( y = 0.86 x + 20 \). The students were then asked to use a graphing calculator to verify that the graph on the board resembled the function equation. A discussion about selecting a calculator view window that would provide a full view of the line and also reflect the blackboard dimensions ensued. This discussion helped students to realize that the view window that would match the blackboard was determined by the horizontal and vertical distances of the established \( x-y \) axes from the edges of the blackboard. A comment supporting the use of negative values for the horizontal distance to the vertical axis on the left (\( X_{\text{min}} \)) and for the distance below the horizontal axis (\( Y_{\text{min}} \)) completed the discussion. The teacher concluded that the students’ term negative distance is not being used as in it is in physics, but instead a negative position is implied. This comment was especially of interest to students who were concurrently taking a physics course. It was also interesting to note that the position of the line was already determined with reference to the \( x-y \) axes, thus no further consideration of the line was needed to have it generated by the graphing technology. Typing the equation into the graphing calculator was sufficient. Students found it rewarding that the technology provided a line similar to that shown on the blackboard.

3. Interpreting the Slope

Just as the tangible tasks of measuring lengths can enhance slope quantification and help students to construct meaning, linking the tasks to applications and function monotonicity can expand student understanding of the slope’s significance.

3.1. Interpretation of the Units of Slope

It was important to note that in the process of algebraically expressing the line, the units of the lengths that constituted the slope were cancelled out, and therefore only the slope magnitude was used. This is the special case of dimensionless linear function representations, which are used widely in mathematics classrooms. Although the alternative of linking slope’s geometric meaning to its interpretation when the axes represent different quantities was not highlighted during this lesson, the following questions could inspire such an extension. For instance, could a speed of \( \frac{5 \text{ m}}{s} \) be perceived as ratio of lengths in which \( \frac{m}{s} \) is viewed in a manner similar to the geometric representation \( \left[ \frac{5 \text{ cm}}{\text{cm}} \right] \frac{m}{s} \)? Would this representation lead the students to correct slope interpretations while simultaneously helping them retain the slope geometrical meaning? It seems that this idea is worth of further research.

3.2. Interpretation of the Slope’s Sign

The lesson also provided opportunities for highlighting the association between the sign of the slope and function increase or decrease. From this point, we use the general slope definition \( \frac{\Delta y}{\Delta x} \), rather than the less formal \( \frac{\text{rise}}{\text{run}} \). In the analyzed example, the value was \( \frac{\Delta y}{\Delta x} = 0.86 \). Solving for \( \Delta y \), we have \( \Delta y = 0.86 \Delta x \). This statement indicates that for every increase in the horizontal distance by, for instance, 1 cm, the vertical height of the line increases by 0.86 cm (\( \Delta y = 0.86(1 \text{ cm}) = 0.86 \text{ cm} \)), and this positive slope causes an increase of the function values. Respectively, if one obtains the slope to be, for example, -2, then this will be interpreted to mean that for every increase of \( x \) by one unit, the \( y \) coordinate of the function decreases by 2 units according to \( \Delta y = -2 \Delta x \). The slope appears as an indicator of how the function’s \( y \)-coordinates change. Consequently, this interpretation will enhance the calculus interpretation of the derivative (often called the slope function by students), which says that if the derivative of \( f(x) \) is negative on a given interval (and therefore the instantaneous slopes are negative), the values of the function will decrease on the interval and vice versa (e.g., see Stewart 2007).
3.3. Example Combining Both Interpretations

When mathematizing the slope, the students realized that the formula \( \frac{\text{rise}}{\text{run}} \) constitutes a ratio of two lengths—the horizontal separation between the selected points and their vertical separation. Seeing the units of lengths in the slope quantification helped initiate students’ transition to thinking in other contexts. The teacher posed the following question: suppose that the slope of a line representing water temperature measured in degrees Fahrenheit versus time over some time interval is 9. Then suppose that the water temperature was labeled on the vertical axis and the time of heating the water, in minutes, was labeled on the horizontal axis. What is the geometric and scientific interpretation of this slope? The students realized that a positive slope indicated a ratio of a positive rise over positive run (it was suggested that run was always considered positive). In a scientific interpretation, it meant that the water temperature was increasing by 9°F for every minute of heating.

3.4. Suggestions for Independent Student Practice

Providing students with other contexts in which the slope can be calculated will broaden the idea and enhance its relevance. The following are some suggested extensions.

1. Use simulations (see, for example, the Balancing Act simulation on the PhET website, [http://phet.colorado.edu/en/simulation/balancing-act](http://phet.colorado.edu/en/simulation/balancing-act)) to copy and paste a few snapshots and have students calculate the slopes and define the corresponding function equations.

2. Provide students with various triangles cut out from a piece of cardboard. Have the students trace the triangles, determine the coordinates, and then construct the mathematical equations of the selected inclinations.

3. Supply students with fixed linear graphs representing various quantities and ask the students to determine function parameters from them.

4. Reflections

This lesson arose from the author’s search to contextualize mathematical ideas and create learning environments according to the contemporary constructivist learning theory. As a novice trying a simple approach, the lesson generated several reflections. Despite being conducted in precalculus classes—where linear functions play a rather marginal role—it turned out to be an engaging and meaningful learning experience for the students and for the teacher. As mathematics concepts are usually presented to students deductively by providing fixed formulas and practicing their applications, this lesson brought in elements of investigation that will be meaningful in the process of function formulation. The students were surprised, for example that \( x-y \) coordinates can be physically measured. Their experience deepened when they observed—on their graphing calculators—a line that resembled the one originally sketched on the board by the teacher. This gave them an evident proof that abstract math concepts are tangible and applicable in reality. By using measurement, the students seemed to find meaning and became encouraged to construct the function on their own.

The question remains: what should be the appropriate grade level to introduce such lesson? It is hypothesized that the most appropriate grade level is when students are introduced to the idea of linear functions for the first time. It is further hypothesized that such a lesson will provide students with a reference for sketching linear functions in their further math courses. The author would like to encourage readers to try conducting this lesson and share their reflections.

References


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Teaching Tricks Can Be Tricky

Linda Fosnaugh*, Patrick Mitchell

Abstract

In this article, a concrete model is used to demonstrate that integer multiplication can be generalized to introduce binomial multiplication, which provides a background for factoring quadratic polynomials. The authors advocate conceptual understanding over rote memorization of procedures and present the shortfalls of the bottoms-up technique.

Keywords: mathematics, education, conceptual understanding, procedural fluency, factoring quadratic polynomials

Procedures and algorithms abound in the mathematics classroom. Throughout their school years, children are taught to manipulate numbers. While drill and practice are important in a math class, an understanding of the underlying concepts is equally important if the student is to retain information and use this knowledge to learn new concepts. According to the National Council of Teachers of Mathematics, a mathematics student must learn with understanding (NCTM, 2000). The student must be able to see the connection between new ideas and prior knowledge of mathematics and should not view mathematics as a series of unrelated topics. Conceptual understanding of mathematics leads to proficiency. Rote memorization of facts and algorithms without understanding leads to confusion as to when to apply a procedure and an inability to solve new problems. These sentiments are reinforced by the National Research Council (NRC) in their report, Adding It Up: Helping Children Learn Mathematics (NRC, 2001). The term mathematical proficiency was introduced by the NRC to describe what it means for a student to learn mathematics successfully (NRC, 2001). This term consists of five interwoven strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. We wish to focus on two of these strands, namely conceptual understanding and procedural fluency. As described by the NRC, conceptual understanding is the comprehension of mathematical concepts, operations, and relations, while procedural fluency refers to the skill in carrying out procedures flexibly, accurately, efficiently and appropriately (NRC, 2001). We will begin our discussion with the development of integer multiplication, binomial multiplication, and factoring quadratic polynomials. We will then demonstrate a “trick” for factoring quadratic polynomials that yields the correct answer but does not develop conceptual understanding.

1. Conceptual Development of Multiplication

A student who has a conceptual understanding of a mathematical concept is able to learn new ideas by connecting them to the previously learned concept. For example, teaching multiplication of binomials using algebra tiles connects to the way that multiplication of two digit numbers is taught using base ten blocks. Consider the visualization of $22 \times 13$ using base ten blocks shown in figure 1.

The representation helps a student visualize...
Teaching Tricks Can Be Tricky

that $22 \times 13 = (20 + 2)(10 + 3)$. That is,

$$
\begin{align*}
20 + 2 & \\
3 \times 2 + 3 & \\
3 \times 20 + 10 & \\
3 \times 20 & \\
10 + 3 & \\
10 \times 2 + 10 & \\
10 & \\
10 & \\
& \\
\end{align*}
$$

This concept carries over to multiplication of binomials. The product $(x + 2)(x + 3)$ is naturally set up using algebra tiles, as illustrated in figure 2.

The student can use and extend their knowledge of integer multiplication to obtain

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6.$$ 

A student who has an understanding of multiplication of binomials with positive constants can then tackle signed values. Such a student knows that

$$(x + 3)(x - 4) = x^2 - 4x + 3x - 12 = x^2 - x - 12,$$

and knows where the constant term comes from. Students can then extend their knowledge to include all binomials and can multiply an expression such as $(3x + 1)(2x - 5)$.

$$(3x + 1)(2x - 5) = 6x^2 - 15x + 2x - 5 = 6x^2 - 13x - 5.$$ 

Notice that the middle term, $-13x$, is the result of adding $3x \times (-5)$ and $1 \times 2x$, and that the factors $3 \times (-5)$ and $1 \times 2$ are also factors of $6 \times (-5)$, the product of the leading and constant terms of the binomial.

2. Factoring Quadratic Polynomials

The NRC advocates teaching conceptual understanding along with drill and practice in order to develop procedural fluency (NRC, 2001). Conceptual understanding helps students remember the procedures practiced in the drill. In addition, the NRC states that students need to know when and how to use the procedures (NRC, 2001). In this section, we illustrate how a student with a conceptual understanding of multiplication of binomials would be prepared to factor a quadratic function by reversing the process.

A student who has developed an understanding of multiplication and factoring and who understands the multiplication procedure knows that

$$(ax + b)(cx + d) = acx^2 + adx + bcx + bd.$$ 

Conceptual and procedural understanding of integer multiplication will help the student find the correct values for $a, b, c,$ and $d$. If that student is faced with factoring the polynomial $6x^2 + 7x - 5$, then the student understands that the term $ac$ is either $6 \times 1$ or $3 \times 2$ (the two factorizations of 6) and the term $bd$ is either $(-1) \times 5$ or $(-5) \times 1$ (the factorizations of $-5$). The student has also learned that the middle term, $7x$, is the result of adding two terms whose coefficients are factors of $6 \times 5$, or 30. In this case, the coefficients are $-3$ and 10.

Using the above information, we obtain

$$6x^2 + 7x - 5 = 6x^2 - 3x + 10x - 5.$$ 

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Notice that the first two terms, $6x^2$ and $-3x$, contain the common factor $3x$, and that the last two terms, $10x$ and $-5$, contain the common factor $5$. Using the technique known as factoring by grouping, the student can factor out the common terms from each pair. That is, $6x^2 - 3x = 3x(2x - 1)$ and $10x - 5 = 5(2x - 1)$. The factorization of $6x^2 + 7x - 5$ is the result of combining these pairs of terms. Thus, factoring by grouping yields

$$6x^2 + 7x - 5 = (6x^2 - 3x) + (10x - 5) = 3x(2x - 1) + 5(2x - 1).$$

This approach to factoring follows naturally from previous knowledge of binomial multiplication and can be useful in generating new insights about factoring higher order polynomials.

3. A Factoring “Trick”

The following method is a popular algorithm for factoring. It is presented under various names such as “bottoms-up factoring” and “slide and divide.” Although the method works and is based on substitution, at first glance the individual steps do not appear to be mathematically justified. This method focuses on a procedural algorithm in lieu of conceptual understanding. Implementation of this method is as follows. Suppose we want to factor the trinomial $6x^2 + 7x - 5$.

**Step 1** Multiply the leading coefficient and constant and replace the constant term with the result. Replace the leading coefficient with 1.

$$x^2 + 7x - 30$$

**Step 2** Factor as you usually would with a leading coefficient of 1.

$$(x + 10)(x - 3)$$

**Step 3** Divide the constants by the original leading coefficient of 6.

$$
\left(x + \frac{10}{6}\right) \left(x - \frac{3}{6}\right)
$$

**Step 4** Simplify rational numbers.

$$
\left(x + \frac{5}{3}\right) \left(x - \frac{1}{2}\right)
$$

**Step 5** “Bottoms up” by multiplying each binomial by the denominator of the constant term.

$$(3x + 5)(2x - 1)$$

Some steps in the above procedure are of great concern. In the first step, the leading coefficient is removed and multiplied by the constant to form a new trinomial. The third step involves dividing each of the constant terms of the binomials by the original leading coefficient. Step five allows us to move the denominator to the position of leading coefficient. Are these steps “legal”? If so, why? Can we use the same method when we encounter other trinomials in another context? In general, these steps change the expression and hence change the problem. Where did this method come from and why does it seem to work?

4. The Trick Revealed

The bottoms-up technique for factoring a quadratic polynomial involves an implicit substitution of variables, which is not appropriate for a beginning algebra student who is learning polynomial factorization. The bottoms-up method relies on a student’s ability to factor expressions of the form $x^2 + bx + c$.

The next logical step is to be able to factor expressions of the form $ax^2 + bx + c$.

Often in mathematics we change the problem into one we are able to solve and then make the appropriate adjustments. In this case we assume that we know how to factor a quadratic polynomial with leading coefficient 1. To change the general quadratic into a form that we can

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_Linda Fosnaugh, Patrick Mitchell_
handle, we perform a change of variable by setting $x = \frac{z}{a}$. Thus we have the following:

$$a \left(\frac{z}{a}\right)^2 + b \left(\frac{z}{a}\right) + c,$$

or

$$\frac{z^2}{a} + \frac{bz}{a} + c.$$

Now multiply the expression by $a$. (We must remember that we did this.)

$$z^2 + bz + ac$$

Note that we have a quadratic polynomial with leading coefficient 1, and thus we can factor it. After factoring the above expression, substitute $z = ax$. Recall that we multiplied the expression by $a$ previously; thus we need to undo that process and divide the expression by $a$.

**Example:** Factor the expression $6x^2 + 7x - 5$.

Substitute $x = \frac{z}{6}$:

$$6 \left(\frac{z}{6}\right)^2 + 7 \left(\frac{z}{6}\right) - 5$$

Simplify:

$$\frac{z^2}{6} + \frac{7z}{6} - 5$$

Multiply by 6:

$$z^2 + 7z - 30$$

Factor:

$$(z + 10)(z - 3)$$

In the bottoms-up method we divided the constant term by 6, reduced the fraction and then moved the denominator to the leading coefficient. In reality, we are just making the following substitution.

Substitute $z = 6x$:

$$(6x + 10)(6x - 3)$$

Factor GCD:

$$2(3x + 5)3(2x - 1)$$

Divide by 6:

$$(3x + 5)(2x - 1)$$

5. Conclusion

The bottoms-up (or slide and divide) method presents a shortcut that hides the underlying mathematics. For example, step 1 tells us to multiply the leading coefficient by the constant term, put that result as the new constant term, and then replace the leading coefficient with 1. It is only when we introduce the change of variables that we see why this series of moves makes sense.

Another point to make is that the variable $x$ in the original problem is not the same as the variable $x$ in step 1 and step 2; however, in steps 3 and 4 it returns to its original meaning. This is performed without any notification to the student. This sort of shortcut is better suited for a magic trick than for a mathematical procedure. It is very important in algebra to realize that, although variables can represent any number, in the context of an algebraic expression or procedure they are fixed. The final step neglects to mention that we are actually multiplying the expression by 6, that is, by the original leading coefficient. The bottoms-up factoring method is mathematically correct, but its justification is above the level of the beginning algebra student. Hence, the method itself appears to work by magic.

Manipulatives and mnemonics that highlight essential steps in mathematical procedures are often used to get our students to reach mathematical proficiency. Our goal as teachers is not to simply find a method that works but rather to establish a conceptual understanding of mathematics. When applying these various techniques, we must ensure that the underlying mathematics is transparent and transportable. The Factoring Quadratic Polynomials section serves as a good example of a technique that is both transparent and transportable. The bottoms-up technique falls short in both categories.

References


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Cracking the Codes of Number Relationships

Lara M. Willox∗, Sarah C. Newman

Abstract

First-grade students investigated number relationships through a series of hands-on activity stations and assessments, gaining experience that altered their perception of numbers.

Keywords: mathematics, education, numeracy, decomposition of numbers, number sense

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1. Overview

All numbers are related. Large numbers may be built through mathematical operations on smaller numbers, which in turn may be built from even smaller numbers. Additionally, there are multiple combinations of smaller numbers that comprise larger ones. According to Van de Walle et al., understanding this part-part-whole relationship of numbers is crucial to developing number sense. They assert that, “To conceptualize a number as being made up of two or more parts is the most important relationship that can be developed about numbers. For example, 7 can be thought of as a set of 3 and a set of 4 or a set of 2 and a set of 5” (Van de Walle et al., 2007). To an adult who has had years of schooling, the idea that seven can be composed in different ways is likely evident, but to a child only beginning to develop number sense, it can seem as new and complex as a foreign language.

Number sense refers to an understanding of the relationships between numbers. Former district mathematics coordinator for Albuquerque Public Schools, Hilde Howden, offers the following definition of number sense:

Number sense can be described as good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms (Howden, 1989).

This article explores the impact of activities using manipulatives on the overall number sense of a class of first-graders. In addition to providing students with a fun and inquiry-based learning environment, the number sense activities also help students to meet state standards and expectations. There are eight overarching expectations for first-grade students outlined in the Common Core Georgia Performance Standards (CCGPS). According to the standards, students will be expected to, “1) make sense of problems and persevere in solving them, 2) reason abstractly and quantitatively, 3) construct viable arguments and critique the reasoning of others, 4) model with mathematics, 5) use appropriate tools strategically, 6) attend to precision, 7) look for and make use of structure, and 8) look for and express regularity in repeated reasoning” (Georgia Department of Education, 2014). These eight expectations need to be met consistently as each standard is addressed.

Many students learn the traditional algorithms for performing basic operations in elementary school, but they may not understand what they are doing or why. They may learn to count, skip-count, add, subtract, even multiply and divide, all without fully comprehending the tasks they are completing. This occurs because students can be taught to calculate the correct answer without understanding the process of obtaining that answer. A student may have

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memorized, for example, that $6 + 4 = 10$, without comprehending the quantities that 6, 4, and 10 represent in this problem. In his 2011 book, Shai Simonson explains what happens when students memorize mathematics without understanding:

> Memorizing mathematics without comprehension is often harmful. If you memorize a poem that you don’t understand, there is still the chance that the flow of the words may have an effect on you. When you memorize dates of historical events, at least you know the chronological order of those events, even if you may not know their significance. When you memorize mathematics without understanding, you delude yourself into thinking that you know something, when in fact you do not. This delusion compounds the lack of understanding; your ability to apply the knowledge, generalize it, or even question its truth is compromised. (Simonson, 2011)

Simonson’s sentiments concerning learning mathematics without comprehension are shared by Kathy Richardson, a nationally renowned mathematics educator who says of such memorization processes, “When children learn to add, subtract, multiply, and divide as a set of procedures, but do not also understand what’s happening to the numbers structurally, they may be able to get the correct answers, but their learning is limited” (Richardson, 2004). To combat such limited learning and ensure students’ success in mastering math concepts, educators must encourage students to explore how numbers are structured and how they function in various settings by teaching them how concepts are linked to procedures.

Experimenting with manipulatives is an excellent way for students to begin constructing such number sense in the primary grades. Manipulatives are simply any concrete objects that students can use to attain knowledge of subject matter through hands-on interaction.

When used appropriately, manipulatives are tools that help students to do their job—learning—more effectively, and to meet the expectations of state grade-level standards.

2. Context

In a first-grade classroom at a Title I school in Georgia, a teacher and student teacher pondered which topic they should choose for a collaborative action research project with a local university. Their assignment was to select a content area in which they believed their students needed to improve. After determining the subject, they worked collaboratively with an assistant professor and a student researcher at the university to develop and execute an intervention plan for growth in the selected content area. At the beginning of the year, the teacher observed that most of her class was unable to fluently compose and decompose numbers to ten, a skill that should be mastered by the end of kindergarten, according to Common Core standards. After observing the students’ work, it was evident to the teacher that the children needed remediation in the area of number sense before they could tackle more complex concepts. To address this need, the teachers decided to select number sense as the area for the research project.

3. Necessary Tools

The teacher and student teacher began collaborating with the research team to find a math intervention for their class. During the initial research phase, the team developed an interest in the work of Kathy Richardson and decided to try her methods and materials. The primary resource they used for selecting and implementing number sense activity centers was Richardson’s 1999 book, *Developing Number Sense: Counting, Comparing, and Pattern*.

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1Common Core Standard MCCCK.OA.3: “Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).”

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the manipulatives associated with it. The manipulatives included color tiles, wooden cubes, number cubes, and Unifix® cubes. They also included what Richardson refers to as “collections,” which are simply groups of ordinary, everyday objects such as paper clips, beans, and rocks (Richardson, 1999). The primary assessment utilized was the AMC (Assessing Math Concepts) Anywhere Hiding Assessment®, which the teacher supplemented with informal assessments of her own. Once the necessary tools had been selected, the students, teachers, and researchers were prepared to embark on an eight month investigation of number relationships.

4. How Many Are Hiding?

The Hiding Assessment® is designed to assess each student’s individual understanding of combinations that comprise the numbers three through ten. The teacher begins by asking a student to select a specified number of objects; for this example we use three. The objects should be identical. Once the student has counted the three requested objects, the teacher initiates a dialogue, which typically resembles the following:

**TEACHER:** “How many color tiles do you have?”

**STUDENT:** “Three.”

**TEACHER:** “That’s exactly right. There are three color tiles. We are going to play a game with these color tiles. I am going to hide some in my hand and show you the rest. I want you to tell me how many I am hiding.”

After this initial conversation, the teacher will hide various subsets of the group of objects and show the student the remaining members until the student has solved all combinations for that number. The process continues through the remaining numbers until the student has mastered each of the numbers from three through ten, or until the student has reached a place where he or she struggles to enumerate the hidden subset accurately or automatically. The number that the student cannot complete becomes his or her instructional level. The numbers that the student will practice independently (through the use of hands-on activities, such as the ones mentioned in the next section) include the number of the instructional level, one number below it and one number above it. So, if a student knows all of the combinations of four, but only some of the combinations of five, then five is that student’s instructional level. The range of numbers that student should practice would be four through six. The student has confidence with parts of four, is ready to learn parts of five, and can practice parts of six for a challenge. Students should be initially assessed to determine their starting instructional level, and they should be reassessed once they have worked on their instructional level for a specified period of time. This allows students to progress to the next level and enables teachers to document their growth and plan future instruction more effectively.

5. Investigating Relationships

Once the teacher and student teacher had discovered and recorded each student’s instructional level, the students could begin their investigation of the numbers three through ten. The students needed to develop the concrete evidence that only self-exploration could provide. To aid students in their investigation, the teacher of this classroom provided a number of hands-on activity stations taken from Richardson’s book (Richardson, 1999). Students used the activity stations in twenty minute increments several times per week. Though game-like in construction, each of the various stations required students to perform mathematical tasks using manipulatives and to record their results. Because different quantities may be used for each activity, the students worked independently and in small groups to investigate different concepts, multiple times, until they were satisfied that they had collected enough evidence to support the idea that all numbers are indeed related.

While several different activity centers were
used, the children had two favorite stations that they visited consistently when investigating their numbers: Number Trains and Number Combination Stories. At the Number Train station, students used the recording sheet that corresponded to their instructional number, two sets of different colored Unifix® cubes (each set containing the instructional number of cubes), and crayons matching the colors of the Unifix® cubes. Using the different colored cubes, students composed their instructional number in a variety of ways and recorded those combinations on their sheet. For example, suppose a student’s instructional level is seven. This student would need the number train sheet for seven, seven Unifix® cubes of one color (blue, for instance), seven Unifix® cubes of another color (red, for instance), a blue crayon and a red crayon. As the student manipulates the Unifix® cubes, he or she notices that seven reds plus zero blues is seven, six blues and one red is seven, one red and six blues is seven, and so on. The student colors the blocks on his or her recording sheet to match the combinations that he or she made with the Unifix® cubes. This process continues until the student has illustrated all possible combinations for the instructional number. (See figures 1 and 2.)

Coming up with as many combinations as possible is a critical part of a child’s learning process. While someone with developed number sense would see that four reds plus three blues is the same as three blues plus four reds, a young student developing number sense sees these as two different statements. Even though $4 + 3$ has the same value as $3 + 4$, educators cannot assume that a student already knows this information. Working with manipulatives, students will discover that these statements are equivalent. They will also notice that regardless of which number they put first and which number they put last (3 or 4 in this case) they still get the same answer, 7.

The Number Train station allowed students to create mathematical statements and to make sense of them. They used their reasoning skills and manipulatives to determine the quantities

Figure 1: A student uses a number train sheet for combinations of ten, working with red and yellow Unifix® cubes.

Figure 2: A sample of a student’s finished work.
behind number symbols and represent them as objects. Students created number train charts, which served as models to help them explain what they did during their center time, justify their conclusions, and remember what they learned.

Students also enjoyed the Number Combination Stories center. In addition to teaching mathematical concepts and introducing students to relevant word problems, this station encourages writing and creative thought. At the Number Combinations Stories station, students began by selecting a background setting such as a garden or an ocean. Next, they select a number of objects equivalent to their instructional number. Once students have completed these steps, they generate math stories based on problems created using parts of their individual number, writing the stories on a recording sheet as they go along. (See figure 3.)

With the introduction of the Common Core State Standards and the increasingly popular idea of “writing across the curriculum,” students are now writing in every subject, including math, validating the need for centers such as this one. Students may use addition or subtraction problems for this activity, but they must represent their work with pictures or manipulatives, and with the traditional algorithm. This allows the teacher to determine whether or not the investigations were conducted thoroughly and correctly.

6. Examining the Impact

After eight months of investigations, the students saw conclusive evidence that numbers are indeed related, and can be created from a host of different combinations. Throughout the period of these investigations, the teacher and student teacher carefully tracked the progress of the students. Their data shows that the majority of the students increased their instructional number by at least two levels. The data does not show that all of the students mastered the ability to work fluently with combinations to ten, but it does show substantial improvement. Samples of student work from centers and independent practice sheets, as well as increased understanding demonstrated in class number talks, provide evidence that the students are laying solid foundations upon which to build further mathematical knowledge. Though all of the students in this classroom participated in the Hiding Assessment®, the graph only shows the data for the fifteen who remained in this classroom throughout the entire school year. Students who joined the class in the middle of the year or changed schools during the course of the school year did not have the opportunity to take the Hiding Assessment® three different times as did their peers, providing inconclusive data concerning their progress.

The teacher and student teacher of this classroom were thrilled by the difference that the activities made in helping their students to become effective “number detectives.” At the beginning of the year, the number sense of many of their students was limited to the numbers 3 through 6. Even though most students did not master every combination for the numbers three through ten, they made significant gains in their understandings of how numbers work as evidenced by our data, which is shown in figure 4.

7. Conclusion

The students were not the only ones challenged by the investigation of number relationships.
relationships. Previously, the teacher had taught mathematics through direct instruction with a methodical approach to computation, focusing on strategies for finding the correct answer, rather than fostering a deep understanding of numbers and their relationships. Relinquishing absolute control of math activities and allowing students to investigate mathematics independently proved difficult for the teacher and student teacher. Instead of being “dictators” during math instruction, they became “facilitators.” At first, it was hard for them to stand back and watch the students choose their own activities, ignoring the overwhelming instinct to tell them what they should work on. As they let the students take responsibility for their learning, however, the teachers were astonished to see students gravitating towards the activities that were right for them, working on the concepts they needed to practice. Having personally experienced success with the stations, both the teacher and student teacher are convinced that the ideas presented in Richardson’s book, when properly implemented, wield the power to instill a deep sense of number relationships in young children.

After the conclusion of our study and the start of the new school year, the assistant professor conducted a follow-up interview with the teacher. She is still using the activity centers in her classroom to guide her new students in obtaining number sense so that they will have a solid foundation on which to build the knowledge of other standard mathematic ideas and principles. As for her former students, they are excelling in mathematics. The second grade teachers report that they are extremely impressed by the number sense exhibited by the first-graders who participated in the number sense building activity stations. They have seen such a difference that they have requested that the first-grade teachers continue to use the number sense activity stations in their classrooms so that their future students will be ready for second grade math when they move up.

References


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iPad Fun: Exploring Slope and Functions Using iPads with the TI-Nspire App

Ann Wheeler*, Brandi Falley

Abstract

In this article, we discuss two iPad-driven projects that can be used in an algebra classroom to teach slope and families of functions. The activities are hands-on and allow students to see the connections of certain careers to mathematics and the beauty of mathematics around them.

Keywords: mathematics, education, slope, function, technology, tablets

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1. Overview

Today’s students thrive in engaging classrooms, where technology abounds and often is vital to the classroom culture. Calculators, computers, and even tablets, such as iPads, have transformed the mathematics classroom into a virtual learning environment of endless possibilities. In addition, national organizations, such as the National Council of Teachers of Mathematics (NCTM), see the importance of technology by listing it as one of the six principles for mathematics that is integral to a mathematics teacher’s instruction (NCTM, 2000).

Besides NCTM’s endorsement of technology, other researchers have also studied the usefulness of such devices in education. Eichenlaub et al. argued that the iPad, though not as integral to academic life as a computer, can be a powerful tool in aiding collaboration, encouraging organization, and assisting learning, regardless of field or level of academic achievement [Eichenlaub et al. 2011]. While many features make the iPad a useful portable computing device, Johnston and Stoll contend that the iPad’s custom applications (apps) provide the leading benefit because they allow manipulation and direct interaction with content [Johnston & Stoll 2011].

Not only are the uses of technology important in the classroom, but the relevance of mathematics to student learning is key to many students’ interests in the lessons. Slope and functions, the mathematical concepts our lessons address, are two concepts that teachers can make applicable to students. For example, one way we related slope to students was through civil engineering: we used the concept of grade or slope to construct various structures, including roads, parking lots, staircases, and wheelchair ramps. We discussed how the steepness of each of these structures influenced their slopes: for example, a steep staircase would have a slope with a greater magnitude than a fairly flat one. For functions, we explored with students the shapes of curves they could see around the school, such as in paintings and architecture. Depending on the object, its general shape might resemble a parabola, an absolute value curve, or something else entirely.

Utilizing these mathematical ideas of slope and functions, coupled with iPad technology and the TI-Nspire calculator, we created two projects for an honors college algebra class during fall 2013. Both activities involved students exploring their surroundings by searching for mathematics in the real world. In the first project, “Investigation Using Slope,” students investigated the mathematics behind the steepness of wheelchair ramps and staircases. For the second project, “Find that Curve!”, students found curves from various families of functions. Not only are these two projects interactive and engaging, they also address Com-
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mon Core State Standards. Sample student work is provided throughout the paper, and both activities are given with TI-Nspire instructions in the Appendices.

2. Project #1: Investigation Using Slope

As a culminating activity for the unit, students utilized iPads with the TI-Nspire App for Project #1: Investigation Using Slope to see how mathematics was a part of their everyday lives (see Appendix A for the complete project, including instructions for the TI-Nspire App). Previously, students learned about the definition of slope and how to calculate its value, given two points or a graph, using the slope formula. For this project, the students reinforced their knowledge about the concept of slope and how to determine its value in various applications. Students were organized into groups of two or three, and each group was given the following materials:

1. the Investigation Using Slope handout,
2. an iPad with the TI-Nspire App,
3. a level, and
4. a tape measure.

Students then walked around campus, searching for two wheelchair ramps and two staircases. Each ramp or staircase needed to be visible enough so that a picture could be taken of its steepness from a level position, as well as accessible enough so that students could measure its steepness with a tape measure. Before students left the classroom, the teacher gave instructions on how to take a two-dimensional looking photo so that no depth was showing (see figures 1 and 2).

In addition, the teacher demonstrated how to use a level so that students could be confident they were taking pictures that were perpendicular to the ground. By taking the extra time to show students how to successfully take photographs with the iPad, the teacher ensured that students produced fairly accurate slope calculations.

Once students completed the picture-taking phase of the assignment, they returned to the

\[\text{Figure 1: Photo of a staircase with exposed depth showing.} \]

\[\text{Figure 2: Photo of a staircase, correctly taken with no exposed depth showing.} \]

\[\text{1CCSS 8.EE.B.5, resp. CCSS HSF-IF.C.7a/b} \]

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classroom to calculate the slope of each wheelchair ramp and staircase using the TI-Nspire App on their iPads. One at a time, students added their photos to the app and created best fit lines to match the slope of each ramp and staircase. They could easily change the slope of their best fit lines by selecting the line with their finger and/or stylus and shifting the line to fit the steepness of the ramp or staircase. Figures 3 and 4, produced by students in class, show the result of using the TI-Nspire App to calculate the best fit lines for a wheelchair ramp and a staircase.

Besides calculating slope with their iPads, we felt it was important for students to also be able to check to see whether their measurements were similar if they completed the measurements with a tape measure. Sixty percent of the student calculations agreed to within one tenth of an inch, with the majority of measurement differences being attributed to difficulties in measuring the slopes accurately.

An additional portion of the assignment required students to compare their measurements to building regulations, which helps students see the mathematics that has to go into structures like staircases and wheelchair ramps we often take for granted. As stated in the activity, both structures must meet certain slope requirements. For example, according to the American Disability Association, wheelchair ramps should have a slope ratio (height to length) of no more than 1:12 (Comforth 2009). Most of the students found that the wheelchair ramps around campus had a slope ratio of roughly 1:12, confirming that our campus ramps met the requirements. Two groups had one set of measurements that were slightly higher than the standard but attributed their calculation differences to difficulties in being able to accurately measure the ramp. For the staircase portion of the assignment, all students found all staircases to meet the standard.

3. Project #2: Find That Curve!

For Find that Curve!, students had already learned about the definition of a function, properties of different functions, and transformations of functions. Similar to the first project, students went outside and engaged in mathematics (see Appendix B for the complete project, including instructions for the TI-Nspire App). Students were once again organized into groups of two or three with each group given a Find that Curve! handout and an iPad with the TI-Nspire App. We gave students descriptions of specific characteristics of curves we wanted them to find across campus. These curves included quadratic, absolute value, square root, and cubic curves.

After students took pictures of the curves with their iPads, they returned to the classroom to determine equations whose graphs approximated the curves in their pictures. Similar to Project #1, students added each photo to their TI-Nspire App and plotted curves, where adjustments could be made to their lines by using their finger and/or
4. Student feedback

Students completed a survey after the projects were completed to provide feedback about the projects. All students indicated that they enjoyed the projects; there were multiple comments made about the positive experiences using iPads and real life applications. One student commented:

I liked becoming aware of the detail put into building slope for stairways and ramps. I also liked using an iPad as a multi-functional tool to take the picture and find the slope.

Another student remarked on the usefulness of the activity to her knowledge of class concepts:

Going out and finding these graphs helped me remember what graphs look like because I had to apply what we learned in class to know what to look for.

Students also mentioned some areas for improvement in the future. Multiple students commented that certain curves in the second project were difficult to find, such as the square root function. Some felt that giving examples of student work would be useful so that they had a clearer idea about what they should be looking for around the university. Even though students struggled to find certain curves, they rose to the challenge and found them without assistance from the teacher.

Along with student feedback, we noted a few limitations along the way. For Investigating Using Slope, one essential requirement was to take pictures that looked two-dimensional, where there was no depth to the staircase and/or ramp. This helped in determining an accurate slope via the TI-Nspire App. However, achieving a perfect two-dimensional image took some practice, which was difficult in the short time allotted for the outdoor investigation portion of the project. Another concern was that the interface caused the iPad slopes
to vary from the actual slopes. Students created best fit lines with their finger and/or stylus, which did not allow for great precision in the measurements. In Project #2, Find That Curve!, the main confounding factor was that pictures were taken from varying perspectives (i.e., taking a picture from below), causing distortion in the images of the curves.

5. Conclusion

Technology in the classroom is always evolving, and we, as educators, need to be comfortable and stay up-to-date with current trends and advances. As we started working with tablets in our classroom, we found that the iPads were user-friendly, and that the students loved to use them. Using iPads with the TI-Nspire App to teach students about slope and families of functions can be a fun and exciting experience for both the teacher and students. With the projects described in this article, we found that the classroom became a more meaningful environment, where students actually saw the mathematics come alive. Students had to search outside the textbook to find the slopes and curves they were studying. We hope in the future to potentially create more iPad-based lessons to engage students in other areas of mathematics, such as gathering real world data and performing statistical measures.

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Appendix A
Project #1: Investigation Using Slope

You will be using the iPad and the TI-Nspire App to investigate the slope of stairs and wheelchair ramps across campus. Take pictures of at least two wheelchair ramps and two staircases, making sure you hold the iPad level and perpendicular to the ramp/staircase. (If you have done this correctly, your level should have the bubble in the middle, and the stairs/wheelchair ramp should look two-dimensional.) After you take a picture, use a tape measure to calculate the height and depth of each ramp/staircase. Record your results in the below table.

<table>
<thead>
<tr>
<th>Object - ramp or staircase</th>
<th>Location on campus</th>
<th>Slope - iPad calculation</th>
<th>Slope - tape measure calculation</th>
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<tbody>
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</tbody>
</table>

1. Were your iPad and tape measure calculations similar? If not, explain why you think there were discrepancies.

2. According to the American Disabilities Association (ADA)\(^1\), a wheelchair ramp should have a slope ratio of no more than 1:12. Do the wheelchair ramps you measured meet this standard? Explain.

3. According to the International Residential Code (IRC)\(^2\) a staircase’s vertical height should be no more than 196 mm with a horizontal length of at least 254 mm. Do the staircases you measured meet this standard? Explain.


Calculating Slope using the iPad with the TI-Nspire App

Directions:
For calculating the slope of a ramp or staircase using your iPad with the TI-Nspire App (version 3.10.0), follow the below directions:

1. Press the TI-Nspire App icon.
2. Press the + sign in the upper left hand corner of your screen.
3. Select “Graphs” from the dropdown menu.
4. Select the camera icon in the upper right hand corner of the screen.
5. Select “Add Photo” from the dropdown menu. (Your selected photo should now be on your screen.)
6. Your cursor should be at the \( f_1(x) = \) line. Make an educated guess as to what you believe is the slope of the ramp or staircase picture you have as a backdrop. (Note: You will need to write an equation of the line, not just the slope, or you will get an error message.) Once you have written an equation, press enter. A blue line with your equation should appear on your screen.
7. Depending on the accuracy of your equation, you may need to adjust your line on the screen. To do so, select the line with your finger and move it to the desired location. (Note: Selecting the line near the origin will result in shifting the y-intercept, whereas selecting the line away from the origin will change the slope.)
8. Once you have the desired equation, record your slope in the table.
9. Email each picture to the instructor.
Appendix B
Project #2: Find That Curve!

Using your iPad with the TI-Nspire App, take pictures of objects around campus that show the types of curves listed below. Then, determine an equation of best fit for the curve in each picture. No “double-dipping” please - each type of curve should have a different object/picture associated with it.

1. A parabolic curve with a positive “a” value, where the formula for a parabola is of the form \( y = a(x - h)^2 + k \)

2. A parabolic curve with a reflection over the x-axis

3. A parabolic curve with a vertical stretch

4. An absolute value curve

5. A square root curve

6. A cubic curve reflected through the origin

After you’ve completed the tasks, take screen captures of your pictures with the overlaid lines of best fit, and send them to the instructor via email.
Determined the Line or Curve of Best Fit using the iPad with the TI-Nspire App

Directions:
For determining the line or curve of best fit using your iPad with the TI-Nspire App (version 3.10.0), follow the below directions:

1. Press the TI-Nspire App icon.
2. Press the + sign in the upper left hand corner of your screen.
3. Select “Graphs” from the dropdown menu.
4. Select the camera icon in the upper right hand corner of the screen.
5. Select “Add Photo” from the dropdown menu. (Your selected photo should now be on your screen.)
6. Your cursor should be at the \( f_1(x) := \) line. Make an educated guess as to what you believe is the equation of the line or curve that you have as a backdrop. Once you have written an equation, press enter. A blue line or curve with your equation should appear on your screen.
7. Depending on the accuracy of your equation, you may need to adjust your curve on the screen. To do so, select the curve with your finger and move it to the desired location. (Note: Selecting points near the origin will result in shifting the \( y \)-intercept, whereas selecting the points away from the origin will change the slope of a line, or the width and orientation of a parabola.)
8. Once you have the desired equation, record your equation on your handout under the appropriate equation description.
9. Email each picture to the instructor.
Follow the Directions
crossword puzzle by Christopher Shaw

Across
1. Mathematica operator
5. Crush
10. Plato may have walked in one
14. Slide—
15. Whales
17. Opener in a group setting?
19. Sluice gate, e.g.
20. One of a standard ten
21. Long ___ of the law
24. McCourt memoir
25. Signs of doom
27. Set of two
29. A la ___
33. ___ Tecnica, source of tech reviews
34. Lug
36. Consumer, of sorts
38. Pythagorean setting
43. Literally, it means "to God"
44. Teen’s tablet
46. Compendium of psyc. disorders
48. Species voiced by Jack Black in an animated film from 2008
51. How many vertices a circle has
52. Drink from a bowl, as a cat
54. Blvd. alternative
56. Suture
57. Strict ending?
58. Fiery morsel
63. Focus about which forces are equalized
68. Conscripts, as a servant
69. Word with bright or right
70. Two-legged attack vehicle employed on Endor
71. Save, as a variable
72. Penalize, as for missing work

Down
1. ictm.org, for instance
2. "A Boy Named ___"
3. Ferrell title role
4. Emeritus faculty, e.g.: Abbr.
5. European language group
6. Almost first lady ___ Heinz Kerry
7. No longer worried by
8. New
9. Livery spread
10. Boat pronoun
11. Traditional royal assistant
12. Many layered things
13. Burden carriers
16. "Comin' ___ the Rye"
18. Seed
21. Super Bowl draw, for some
22. Rhine tributary
23. It was once French Sudan
26. Word with error or victory
28. Conquistador Vasco
30. Old show
31. Mai ___
32. Zeta theta link
35. Mathematics thesis result, perhaps
37. Terminus
39. Nib
40. Informal academic gathering
41. Landlocked Asian land
42. Coastal flier
45. Wood used for bowmaking
46. Wouldn't dream of
47. Acts unmisgrily
49. Hot water
50. Disinclined
52. St. ___ (Caribbean isle)
53. ___ and now
55. Pitcher’s stat.
59. Liberal ___ college
60. Boxing matchup
61. Big ‘do
62. "Metamorphoses" poet
64. ___ Offensive
65. Union sealing proclamation
66. Gumshoe
67. Prattle