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From the Editors…

Welcome to the Fall (almost winter) issue of the *Illinois Mathematics Teacher*. We hope your school year is going well. In this issue you will notice that the first three articles are reprinted from the last issue. We owe a big apology to co-authors that were not listed with the original printing of the articles.

This issue of the *IMT* once again contains many useful and interesting articles and activities covering a wide range of topics. There are several articles that deal with middle or high school level material. Andrew Samide’s geometry class wrote an article about a geometry homework problem that lead to a class exploration. Denise Reid and Janice Lowe created a word find and a crossword puzzle using geometry terms. David Duncan and Bonnie Litwiller have submitted a problem that connects math to the real world.

We are also fortunate to have several articles on topics of general interest. Ariel Ramirez’s article on calculators should get many of us thinking about how we use calculators in the classroom. Barbara O’Donnell has an interesting article about problem solving. Finally, Mary Thomas has included an article with some quick ideas you can use in your classroom.

We would appreciate hearing from any of you out there who are reading the journal. We would especially like to hear about any activities from the *IMT* that you have used with your students. Your comments and constructive criticism are heartily solicited.

Please consider submitting an article or classroom activity to the *IMT*. Your articles are needed to continue the sharing of ideas and the publishing of this journal.

Thank you for sharing.

*Marilyn and Tammy* editors
Teachers Helping Teachers: 
Words of Wisdom from New Mathematics Teachers

Gay Ragan  P. Mark Taylor
gayragan@smsu.edu  Knoxville, TN
Springfield, MO

Jennifer Hodits  Melissa Helmreich
Jefferson City, MO  Columbia, MO

Whether you have been teaching for decades or you are just beginning to student teach, teachers should be life-long learners. With each new course you teach, there are obvious lessons to learn about both the content and pedagogy involved. You may have been teaching a course for decades and yet found new ways to teach a concept in order to introduce mathematics to diverse learners. As Claude Fuess said, after 40 years of teaching, “I was still learning when I taught the last class” (as cited in Hurst & Redding, 1999, p. 71).

Do you remember the incredible amount you learned during student teaching and your first year of mathematics teaching? Learning the rhythm of the school day from the teacher’s perspective; learning how to teach lessons; learning how to make assignments (and find time to get it all graded), and learning how to still have a personal life. Most of us would look back and agree that there was a lot left to learn after that early point in our teaching career and at the same time, there was a lot that had been learned.

In particular, a beginning teacher’s success begins well before they enter the classroom and is sustained with the help of others. According to the National Council of Teachers of Mathematics (NCTM), “…beginning teachers need the professional support of administrators and more experienced educators...[and] …believe that school systems and universities must assume shared responsibility for [such] support of beginning teachers…” (NCTM, 2002, p. 4). In response, we asked students finishing their student teaching to write letters offering advice to the next cohort of student teachers and after their first year of teaching mathematics in high schools, we asked them to write another letter, this time giving advice to first-year mathematics teachers. At that early stage in a teaching career, what words of wisdom could these mathematics teachers offer to those who are following them in the teaching profession? Could they possibly have words of wisdom to offer their more experienced colleagues? We believe their responses provide professional support to teachers regardless of experience level.

Given wide latitude of what topics to address, these new teachers’ pearls of wisdom tended to address one of three broad categories: students, teachers, and the personal aspects of teaching. The comments of two such mathematics teachers, unedited, are included as illustrations of the insight that teachers can have early in their careers.

**Students**

**Motivation**

The students come into class carrying more baggage (and I am not referring to their back-packs) than I could ever imagine, and as their teachers, we need to remember that mathematics may not be what is on their mind or even most important to them that day. Consequently, as a teacher, I have come to understand that motivating students to learn is essential. The first thing that student’s notice is whether you want to be at school. If you are excited to be there, the excitement rubs off...
on your students. If you love what you are doing, it shows. It typically seems that students either love or hate mathematics. So if you have fun while teaching mathematics, the kids will start to love mathematics too (or at least smile because they think you are so goofy). The teacher has to understand that mathematics may not be the most important thing to the student today so we have to motivate them to want to learn. Students need to know that you want them to be there. If you care about them, they are more likely to care about the class. Students treat you with the same amount of respect that you give them.

Classroom Management

The most important thing I learned during student teaching was classroom management skills. You never get to really practice classroom management strategies in pre-service methods courses. My cooperating teacher had a lot of good ideas about classroom management and I incorporated the following techniques. I outlined the classroom rules. My policy is to have as few of rules as possible. With numerous rules, you need to have numerous disciplinary actions laid out. Thus, I usually only have two or three rules, such as respecting the teacher and classmates and to come to class prepared. Also before the first day of school, you should receive a list of students who are enrolled in your class. I use this list to set up an alphabetical seating chart so I can learn their names. I also show the list to teachers from last year so they can point out students who will be helpful and those I need to keep a close eye on!

I would also advise you to be prepared for the first time a student confronts you. The first time a student was rude to me and told me that I was not a good teacher I got upset, but now I realize I should not have taken this personally.

As a result, I have come to believe that classroom management is just another thing that will come as I gain more teaching experience.

Parents

Another thing that you will need to be prepared to do is address a parent. Communicating with parents throughout the year is a good habit to form. I try to contact parents at the beginning of the year, either by telephone, mail, or e-mail, just to introduce myself and provide them with information about how to contact me.

Parent support is important. I try to call ALL of my students' parents or guardians after the first test. Parents love to hear good news about their child such as their child “aced” a test. Parents are usually surprised to receive a good call—either their child is always good so they never get a call or they only get bad telephone calls. Kids really love calls home (for good news) too. It helps motivate students to learn when they know that you are proud of them. Many parents appreciate the information and later, when you need to call with a problem, they know who you are and are more likely to help solve the problem.

Teachers

Logistics of Teaching

Due to the fact that I did my teaching in the spring, my teacher had already set the tone of the classroom. My first week of student teaching went well because students were taking first semester final examinations. This provided me with a great opportunity to learn the students’ names by looking at the seating chart while they took the examination. I had made arrangements the semester prior to observe
the teacher whom I had been assigned to complete my student teaching internship with, so I became familiar with this teacher’s daily schedule and classroom procedures. I would recommend you do this once you are notified of your student teaching placement. The week after final examinations were administered, my partner teacher had me teach “parts” of some lessons. Some days she would ask me to review certain problems an hour before the class began or ask me to change problems I had prepared to use in my lesson, so you just have to be willing to try things and be flexible.

Yet, little did I know that my first job teaching would not begin so smoothly. Initially, I felt as if I was bombarded with so much information about school procedures that I would never remember it all. I quickly found that I did not have to memorize everything before the first day of school. However, there are a few things you may want to have decided before the first day of school. In each class syllabus, I outlined for the students the assignments that we will be doing in class. This included not only homework and tests, but also any projects, portfolios, and/or notebooks. The syllabus also showed how each assignment will be graded.

Lesson plans and grading will be the two hardest things to deal with on a daily basis that last all year long and for the rest of your teaching career. I try to have plans for at least three days in a row. Then, if you have a lot of grading one night, have to attend a school function, or just don’t feel like doing anything after a hard day, you are still prepared for the following day. Organization is a key to a successful second year. Keeping a three-ring-binder or file system that contains lesson plans, worksheets, and tests from your first year come in very handy the next year. More than likely, you will teach the same class during your second year, and thus can use items from last year. This cuts down on planning time tremendously, but not completely. Also with your lesson plans, keep notes of how the lesson went and what you may need to change for the next year. You need to reflect on each lesson and modify it accordingly. Keep in mind that each class is unique. As far as grading is concerned, you need to find a system that works for you. Personally, I always wanted to know how I had done on homework and especially tests as soon as possible. Feedback can also help the students learn. Thus, I try to keep up with grading, and not let it pile up on my desk. This may be easier for some content areas, so I just do the best I can. Students want to receive credit for everything so rather than grading each assignment for every student, I either check for completeness or give a homework check where students write down answers to select questions. Some teachers even require students to keep a notebook of every assignment and then give a homework check before a unit test.

Collaboration

I would advise you to go into teaching with an open-mind and be flexible. During student teaching, your partner teacher probably has procedures in their classroom established, so you need to be willing to adapt to their style. It is not that you cannot try your own things, but you need to keep the stability for the students’ sake.

My partner teacher was great about helping me plan lessons. After the first few weeks, I began teaching one of the mathematics classes on my own but my teacher always asked me what I had planned and gave me suggestions. One thing I always remembered was that these were still her classes and even though she was willing
to let me try things, she had the students’ learning in mind. Therefore, when she would make changes to my lesson plans, I knew it was not something I did wrong, and rather it was an opportunity for me to learn from her teaching experience.

During your first year of teaching, a mentor teacher is a great resource that should be utilized. My mentor teacher was always very willing to share tips on how to deal with many issues. The key is to ask questions constantly! Remember, these veteran teachers know what it is like to be a first-year teacher and are willing to share their experiences to help in any way possible.

Both during your student teaching and first year of teaching, try and visit other teacher’s classrooms too. Observe teachers who teach different subjects or grade levels. Every teacher has their own style of teaching—you can learn a little something from everyone and gain different ideas to use in your classroom. Again, “steal” ideas from other teachers and find what works best for you.

Teaching is a time to learn. I learned a lot about myself, both as a student teacher and as a beginning teacher, by trying many strategies in the classroom, reflecting on my teaching, and learning from other teachers.

Personal Aspects

Once you begin your student teaching internship, enjoy every minute because the next sixteen weeks will go by fast. Student teaching takes up more time that I could ever have guessed. Plan your schedule so that you will have plenty of time to devote to your student teaching (and yourself). It will make everything go much smoother if you have the time to prepare your lessons. Your days will be very long, and mornings will come early. There will be days you don’t want to get out of bed. I suggest that you think about that one student in your class who is always there willing to learn or who always helps you by answering questions during discussions. Students like this make every day worthwhile!

You have probably heard that your first year of teaching is going to be the hardest year of your career. While this may be true, I believe that there are ways to make some things a little easier to deal with. The first thing I want you to know is that if you are anything like me—RELAX, take a deep breath, and have a great time. The most important thing you can do is to have fun, and let the students see you having fun. Take in as much as you can from the teachers around you, and most importantly, enjoy yourself.

Mathematics: The Missing Topic

The lessons these teachers have learned and chosen to pass on in their letters of advice reflect the kind of learning and growth that all teachers should have. The things that these mathematics teachers have chosen to pass on are the things that they have had to pay the most attention to in their first years of teaching. They are also the things that teacher educators often throw their hands up into the air and say “that is something that only comes with experience.” Unfortunately, this has its price.

In August 2002, the NCTM adopted a position statement on the induction and mentoring of new teachers recommending that “school systems and universities…provide [beginning teachers] with opportunities for further development of mathematics content, pedagogy, and management strategies” (NCTM, 2002, p. 4). Interestingly, we noticed that their letters did address many issues related to management but less on pedagogy and nonexistent in these letters was a focused
discussion on mathematics and the mathematics as the students understand it. These teachers have successfully completed a lot of mathematics courses at the college level and have taken at least one course on how to teach mathematics. The mathematics, however, often gets overshadowed in the initial teaching years by the newness and intensity of the experience of operating in the school culture as a teacher for the first time. This points to an important goal for teacher educators: How can we rework teacher education to help new mathematics teachers to stay focused on the mathematics that the students are learning?

The following suggestions are offered not as “the correct solutions” but merely a place to start the brainstorming on how to solve this dilemma. One possible way to keep the mathematics from being overshadowed is to remove some of the intensity from the student teaching experience. This could be accomplished, as it is in some universities, by having an extended internship experience that does not have them teach a full load. This offers time for them to reflect deeply on the mathematics they are teaching as the students are coming to understand it. An example of this would be the program at the University of Tennessee (UT), where the internship is a full year, with students taking one or two courses during the first semester and another course or two in the spring. In addition, interns take courses during that time including, analysis of teaching, mathematics methods, and action research. This gives the mathematics teacher educators at UT several opportunities to keep the interns focused on mathematics.

Another possibility would be to introduce them into the school culture and the teacher’s role sooner. This is a concept that many colleges of education are trying to accomplish through earlier and more frequent field experiences. Whatever the remedy turns out to be, the hope is that eventually our young teachers advice to those who follow will be more mathematical.

Conclusion

The words of wisdom offered by these new mathematics teachers hold promise and warning simultaneously. They are learning important lessons about motivating students, how to interact with parents, and how to be co-learners in the school environment. These words of wisdom are all invaluable and mathematics teachers of any experience level should heed the advice given by these two individuals. At the same time, we need to keep in mind that we are teachers of mathematics and that the content should be questioned and thought out. The curricular and instructional choices made by teachers every day, often taken for granted, are critical to the students’ opportunity to learn. The assessment of students’ mathematical understanding and skills are crucial and should receive a lot of the teachers’ attention. This, too, is true for new mathematics teachers as well as those with 30 years of experience. We conclude with these final words of wisdom from two mathematics teachers:

The most important lessons we learned from our teaching experiences is to be yourself, have fun, and never forget that it is all about the students. The main reasons we come to school every morning, is for the students. What is best for your students is what you always need to keep at the front of your mind. Your first years of teaching go by quickly. You will survive and before you know it, you will have words of wisdom to share with beginning teachers.
References


Guidelines for Submitting Manuscripts

Prospective authors should send:

- **Five (5) copies of your article**, typed, double-spaced, 1-inch margins. Put your name and address only in the cover letter. No identifying information should be contained in copies of the manuscript. Articles should be no more than ten pages in length, including any graphics or supplementary materials.

- **A diskette with your article, including any graphics**. We prefer that the article be written in Microsoft Word and that it be saved on an IBM-compatible disk. Graphics should be computer-generated or drawn in black ink and fit on an 8 1/2”×11” page.

- **Your name, address, phone, and e-mail** (if available) should be included in a cover letter.

- **A photo of yourself (Illinois authors only)**, color or black/white.

To: Marilyn Hasty and Tammy Voepel
Mathematics Department
Box 1653
Southern Illinois University Edwardsville
Edwardsville, IL 62026-1653
Pigs in the Classroom?
Claire Krukenberg, Joyce Bishop, & Wendi Chenault (pictured)
Eastern Illinois University
Charleston, IL 61920

Using games to involve students in learning activities is a tried-and-true educational practice, one that preservice teachers enthusiastically endorse. A recent presentation by Dr. Claire Krukenberg to members of Math Energy, an Eastern Illinois University student organization for students preparing to be elementary, middle level, or special education students, capitalized on this enthusiasm for games by incorporating the “Pass the Pigs™” game. One goal was to explore ideas of probability in a fun way that was accessible to people with a range of ability levels. Another goal was to encourage students to consider how subtle changes in game rules affect the probabilities embedded in the activities and consequently, how the changes might affect strategy.

In this article we first discuss the activity as it was presented to 150 members of Math Energy. Then we share one student’s response to the challenge at the end of the presentation to try the activity with some children in her life. Finally, we discuss some of the information we have collected about “Pass the Pigs” that might be useful to classroom teachers and some of the questions we have raised.

The presentation was based on the following assumptions:

1) As a teacher your goal should be to have all students surpass you in knowledge.

2) Kids are very creative, usually eager, and are able to learn faster than we let them.

We can set the stage for students to stretch their knowledge and think creatively by setting up game situations in which the students must make up their own rules. The astute teacher then must be ready to seize the moment, have fun, and learn.

The Math Energy program began with a standard classroom manipulative – a single die, and the discussion unfolded. What can we do with a single die? Some possibilities follow, and you are encouraged to consider how the activities might be adapted to a range of grade levels. Let’s start with the simplest: What are you going to do with one die and one child? Toss the die – then what? Think about some simple ways to record results. We could match results with pictures of die faces, or with numerals, 1, 2, 3, 4, 5, or with words, “One, two, three, four, five”. We could toss the die again – Did we get a number that matches the number we rolled, or is it less or more? We can challenge students to think deeply about the mathematics with questions such as: How many tosses does it take to match your first toss? How many tosses will it take to get all six different sides? Is it certain that we will match the first toss? Is it certain that we will get all six different sides?

We can sneakily incorporate addition – what was the sum of the first two tosses, or three, or . . . .? Advanced students could be challenged to express the sum in base 6. For example, consider five tosses resulting in 6, 5, 3, 4, 1. The next step might be to regroup to 6, 6, 3, 4 and then to 6, 6, 6, 1 and then to 316. Each step could be modeled with dice or with diagrams.
This is a productive line of thought, but we believe that children learn well from interactions with others, so let’s move on to activities with two or more people. For two players we could ask whether it makes a difference whether two people toss the die at the same time or to take turns. Let’s look at some possibilities and the issues they raise:

1) Suppose you play to see who gets the bigger number (or smaller). How do you score? How do you score a tie? Do you score one point for a win, or a numerical value representing the difference in tosses? Very young children can begin with equal numbers of chips and for each set of tosses the loser must give the winner a chip.

2) How many tosses are needed to repeat the first toss? You can explore this individually, or in pairs, or with the whole class. Individual students can record their first roll, and then roll again until they match their first roll. Or the teacher or a student can roll and then each student rolls until he or she gets a match with the teacher, keeping count of how many rolls it takes. Or pairs of students can compete. Students can determine the rules. Repeat this several times to develop a sense of what the expected value is.

3) A variation on 2) is to roll until you match a previous roll. For example, in tosses of 1, 4, 3, 2, 4, the student hasn’t matched the first roll yet, but has repeated the 4. Students could keep track of the number of rolls it takes to get a repeat.

A possible writing assignment related to these activities could be: You get four rolls to try to either match your first roll or roll a repeat and why? What if you could have a different number of rolls, what number would you choose and why? This line of questioning encourages students to discover that although at least one repeat must occur in seven rolls, it is theoretically possible that one might never match the first roll.

Adding another option introduces opportunities for new strategies. Suppose Player A tosses and then Player B chooses to toss or pass. Winning the toss would be determined by who has the larger number. If B’s number is greater than A’s then B wins. If B’s number is smaller than A’s, then A wins. Note that we haven’t addressed the issue of a tie. Possible scores for a tie include: A wins, B wins, nobody wins, or both get a point. A sample writing assignment based on this activity might now be: Player A tosses a 3. Should player B toss or concede? Is your answer affected by the rule for scoring a tie? Would your strategy change if Player A tossed a 4? Support your answer. Another writing prompt could be: Suppose the first roll is a 4. Would you rather be Player A or Player B? Explain your reasoning.

By changing the payoff for various outcomes, we raise some new issues. Consider this variation: 1. A tosses first. If B rolls a bigger number than A, then B scores 2. If B rolls a smaller number, then A scores 2, If B rolls the same number or passes, then B scores zero. A third scoring scheme would be to have B score 1 for a tie and zero for a pass. A fourth scoring scheme would be for B to score zero for a tie and for A to score one if B passes. Possible writing questions: If A rolls a 3, what strategy would you follow and why? How would your strategy differ from scoring scheme 1 to scheme 2 or scheme 3? Explain how and why.

Understandably, we would expect totally different kinds of responses to these questions from different age levels. Young
children just enjoy playing the game, but over time, they develop strategy. At lower grade levels students can develop “a feeling” about probability as they play the games. This is valuable because our society does not provide many opportunities for people to develop mathematical intuitions about probabilistic situations. With these kinds of experiences under their belts, students will be better prepared to make sense at higher grade levels of the mathematical concepts of probability, odds, and expected value. Note that by minor changes, a large number of variations can be introduced. Each provides opportunity for play, for data collection, and for written assignments about results or strategies.

So far, our activities center on an idealized situation, a fair die, which makes it easy to compare the probabilities of many outcomes. Unfortunately, real life is not so tidy, and so we introduce a less tidy alternative: we “Bring Out the Pigs!”

In 2003, ICTM highlighted a game called “Pass the Pigs.” The game is delightful, with application in arithmetic and strategy at early to adult levels. It is based on tossing two pigs. These pigs can end up on their side (Sider), feet (Trotter), back (Razorback), snout (Snouter), or snout and ear simultaneously (Leaning Jowler).

To play the official game, one player tosses the two pigs and the toss is scored as follows:

- Pig Out (pigs on opposite sides)
  - Lose all points on that round

- Sider (pigs on same sides)
  - 1 point

- Trotter
  - 5 points, double Trotter 20 points

- Razorback
  - 5 points, double Razorback 20 points

- Snouter
  - 10 points, double Snouter 40 points

- Leaning Jowler
  - 15 points, double Leaning Jowler 60 points

- Oinker (one against the other)
  - Back to zero for the game

- Mixed Combo
  - add combined score for any combination of Trotter, Razorback, Leaning Jowler, or Snouter

The player then may choose to continue his turn and toss the pigs again, or quit, bank his score, and pass his pigs to the next player. Usually a target is set for a winning score.

At this point in the Math Energy program, Pass the Pig™ games were distributed to the students in attendance. Rules were established, and pigs started rolling. Data of outcomes were collected but time constraints did not allow a thorough investigation of all the possible questions. As a final challenge, students were invited to take some pigs into a classroom and, without telling the children how the official game is played, notice what ideas they produced, and then submit a report. Wendi Chenault, who was enrolled in a mathematics course for elementary majors, rose to the challenge, and her account follows.

For a new activity for my pigs I took them and let them play with my daughter’s second grade class. Many of the students thought of using them as pieces on a game board and some others thought up games that involved throwing the pigs. The idea was to make a wall out of Legos™ or paper and throw the pigs at the wall to see which pig made the biggest hole. One student
wanted to make a circle of Pokemon™ cards and then toss the pigs on the cards. What ever card the pig landed on would be the card you would use to play with. He also said that if he was just trading cards with someone they would toss the pigs on each others’ cards to see which cards they would trade. Another child suggested a game that she called “Who Lives” where she drew up a game board with many obstacles along the way and two people would roll the dice and move the pigs that many spaces. Each space had different things written on it such as “Go back 3 spaces,” “You die.” “Go to the alligator,” etc. One young man came up with the idea of labeling each side of the pig with a number 1 through 6. The side with one spot was one, no spots was two, and on its back was 4, because of 4 legs. Three would be if it landed on its feet. Five would be landing on the snout and an ear [Leaning Jowler in the game directions] and six would be landing on the snout only. They decided that they would have a straight game board with a certain amount of spots on it. Then they would use some other markers on this board to mark the places where they were. Each person would have a chance to roll but they have to pick pigs or dice before the game started. When they rolled they would move that many spaces. Whoever reached the end first would be the winner. A twist on the game would be to give the pigs higher values on the harder-to-get rolls. It would be more of an incentive to choose the pigs over the dice.

I believe that this is a good lesson for the children because as we found out in Math Energy, the pigs don’t always roll in a certain way. To better explain this I can use the example of a die, if you roll a die you have a one-in-six chance of getting a six. With the pigs when you roll them you are more likely to get a pig to land on its side rather than its snout. This is because of how the pigs are shaped as compared to how the dice are shaped. A die is equal on all sides where as the pigs are not.

The children in this second-grade class devised several imaginative ways to use the pigs. Many of them apparently associated the pigs with the tokens they had used for other games. One perceptive young man recognized that the pigs could be used to generate different values similar to the manner in which dice are often used in games. It would be interesting to know how much he understood about the distinctions between equally likely outcomes such as the possibilities for rolling a single die and unequally likely outcomes such as the possibilities for tossing a pig. The children were clearly enthusiastic about pigs, an important first step toward encouraging them to think creatively.

These varied experiences point to the educational possibilities inherent in rolling pigs. The positions in which the pigs might land are not equally likely, and who knows whether a side is more likely than a back? This incongruity sets most of our associations akimbo. We have no numbers with their attached numerical order and no equally likely outcomes, so what happens to all our previous ideas about probability when we throw out the dice and bring in the pigs? How many of the dice activities could
be adapted to the pigs, and what changes or adaptations would be needed?

To parallel many of the previous games mentioned, it would be useful to have a feel for the distribution of the various outcomes. We wondered how the different outcomes would be distributed. Two EIU classes of preservice teachers conducted an experiment in which pairs of students tossed single pigs 100 times. To examine whether there was much variation among individual pigs, each pair traded pigs at the 50 mark and kept separate totals. Their results indicated that differences between pigs were negligible. Their results are included in Table 1 at the end of the article.

Our figures are based on a sample of 2218. Would another trial produce similar results? Try it with your class. What if you had a sample of 10 instead of 2218? Would 50 be enough? 5000? These questions raise sampling issues that students need to consider.

Other issues arise. Was our sample large enough to provide the basis for a conclusion about the probabilities of the different outcomes? Does the surface on which the pigs are rolled make a difference? Does it make a difference whether you drop the pigs or roll the pigs? In the messy real world, issues like these must be considered.

With dice, all outcomes are equally likely and the numbers of dots provide an order for the outcomes, for instance, three is greater than two. In single pig games, however, should a Snouter beat a Razorback? In the matching games, would you rather try to match a Sider or a Trotter? In two pig games, are the assignment of points in the commercial game “Pass the Pigs™” a realistic assignment of points? If not, what affect does this have on the strategies used to play the game? The pigs’ unequally likely outcomes force us to rethink ideas of probability that we don’t have to consider with fair dice. That makes them useful for helping students develop a deeper understanding of the concepts of probability. Caution! These considerations are not intended to detract from enjoyment of the game.

To help you think about these questions, Table 2 at the end of the article has the two-pig results extrapolated from the classroom data. This data was derived from single pig results. The astute observer will notice that the possibility of an Oinker has been ignored.

To find the “Pass the Pigs” game, search the internet for “Pass the Pigs game.” A Yahoo search yielded over 351,000 entries with several familiar sources on the first 10 pages.

Why don’t you roll out the dice or the pigs with your class and see what questions and answers they raise for you? Let us know what happens. This could include concise information about activities and outcomes, or tabular information of additional data. Send information and data to Claire Krukenberg, 2531 Village Road, Charleston, IL, 61920-4233.

Note: The authors are indebted to Dr. Joan Henn, who arranged for the presentation and later was instrumental in collecting data in her classes.
Table 1

<table>
<thead>
<tr>
<th>Position</th>
<th>Trial 1</th>
<th>% Trial 1</th>
<th>Trial 2</th>
<th>% Trial 2</th>
<th>Total</th>
<th>% Total</th>
</tr>
</thead>
<tbody>
<tr>
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<td>36.29</td>
<td>362</td>
<td>32.85</td>
<td>767</td>
<td>34.58</td>
</tr>
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<td>Side (dot)</td>
<td>327</td>
<td>29.30</td>
<td>346</td>
<td>31.40</td>
<td>673</td>
<td>30.34</td>
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Table 2

<table>
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<th>Results for Tossing Two Pigs (two unhappy pigs)</th>
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<td>Position</td>
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<td>----------</td>
</tr>
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<tr>
<td>Double Learning Jowler</td>
</tr>
<tr>
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</tr>
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</table>
Developing Measurement Concepts with Young Children

Kim Hartweg & Melissa Hardy
kk-hartweg@wiu.edu
Macomb, IL 61455

“I’m bigger than you!” “I have more red bears than blue bears.” “I didn’t get as much popcorn as he did!” These comments, or a variation of them, are common occurrences in a pre-kindergarten classroom. Everyday, pre-kindergarten children are demonstrating knowledge about mathematics. They may be comparing, counting, measuring, or participating in many other mathematical activities. They use math in their play; they use math when they eat; they use math when they sing songs. It is difficult to keep mathematics out of the classroom. Yet, not all mathematical concepts are easy to teach children. The basic ideas related to measurement may be obvious to an adult, but they have to be developed and taught to children. As children encounter and apply these ideas in an informal and intuitive way, their generalizations and verbalizations advance (Liedtke, 1990). Learning to measure cannot be done through listening or paper and pencil activities, and although young children are still developing foundational skills, it is important to begin measurement activities early in the primary grades (Wilson & Rowland, 1993).

To lay the foundation for measurement, children should take part in activities focused on comparing, such as comparing objects perceptually, comparing them directly, and comparing them indirectly (Reys et al., 1998). This concept helps children gain understanding of particular attributes and the associated vocabulary, while learning the procedures that will help them assign a number to a measurement. “Children come to school with some concept of length and some vocabulary associated with it. However, they often have what adults may consider misconceptions about length. These misconceptions disappear as children develop cognitively and are given constructive experiences” (Reys et al., 1998). Piagetians claimed that children achieved an understanding of measurement only at about age 9 (Clements, 1999). But more recent studies have shown that children can achieve measurement concepts at a much younger age than previously suggested (Wilson & Rowland, 1993). Even preschoolers can compare two objects directly and recognize equality or inequality of length (Boulton-Lewis, Wilss, & Mutch, 1996).

A Pattern Lesson Develops Measurement Concepts

The ability of preschoolers to compare objects and recognize equality or inequality of length became evident when an introductory lesson on patterns was presented to a group of 4 and 5 year olds. The lesson was adapted from the Investigations in Number, Data, and Space kindergarten curriculum. The lesson began with the reading of the story book Luka’s Quilt by Georgia Guback. The story describes the relationship between Luka and her grandmother as a traditional two-color Hawaiian quilt is created for Luka. Luka quarrels with her grandmother over the use of only two colors in the quilt. Eventually a truce is declared and a compromise is made to please both Luka and her grandmother.

After reading Luka’s Quilt and discussing the use of two colors or lots of
colors to create patterns in a quilt, the preschool children were asked to use unifix cubes to build trains that were 8 cubes long using colors of their choice. The teacher and aids watched students with the following questions in mind: Can students make trains of 8? What type of arrangements do they make? Can students describe their trains? Are students looking for particular colors?

During the student work time it became apparent that measurement concepts were also being developed during this lesson as one student excitedly exclaimed, “Look, you don’t even have to count. You can just hold up this train next to this train [a train of 8 unifix cubes] to see if it’s the same.” Pre-school students in this group began to use this measurement technique rather than count 1-8 for each train they created.

After students created a variety of trains, the teacher told students to pick their favorite train to share with the class. As students shared their train with the other students they told which colors they picked and how they knew there were 8 cubes. As students surveyed the class trains, the teacher asked questions such as: What do you notice about the different trains? Which trains make it easy to tell what color comes next? Finally, the class sorted the trains into one of two groups, a pattern group and a non-pattern group, as they continued to discuss what was the same and different about the trains in each group. While the focus of this lesson was originally intended to focus on learning about patterns, the ability of pre-school students to recognize the attribute of length and to compare the trains according to this attribute showed that measurement concepts can and should be introduced at a young age.

The Teacher’s Role in Developing Concepts

Teachers can influence the development of mathematical concepts, such as measurement and pattern relations. Providing young children activities and problems that promote curiosity and exploration will help build a solid mathematical foundation and may influence their study of mathematics in future years. Specific websites that could be used to support pattern activities as found in the Luka’s Quilt lesson and websites that support the teaching of measurement and estimation for young children used are provided below.

Quilt Maker
http://www.licm.com/shtm_content/s_quilt.shtml

Complete the Pattern
http://matti.usu.edu/nlvm/nav/frames_asid_184_g_1_t_1.html

Hawaiian Quilts
http://www.quiltshawaii.com

Lessons PreK-2
http://illuminations.nctm.org

Introduction to Measurement for Primary Students
http://mathforum.org/varnelle/krods.html

Non-standard Measuring
http://mathforum.org/paths/measurement/nonstand.html

Sand Babies
http://www.pbs.org/teachersource/mathline/lessonplans/esmp/sandbabies/sandbabies_procedure.shtm

Learning Geometry and Measurement Concepts by Creating Paths and Navigating Mazes: Hiding Ladybug
References


Cahill’s Conjecture

Andrew Samide’s 7/8 Geometry Class
Montini Catholic High School
Lombard, IL  60148

Cahill’s conjecture turns into an interesting investigation of mathematics. It was derived after he (a student in class) presented his solution to a problem that he was assigned for homework.

The Problem

\[ \begin{align*}
A & \quad B \\
C & \quad D
\end{align*} \]

Triangles \( ABC \) and \( DBC \) are isosceles. \( AB = AC \) and \( DC = DB \). Find the values of \( x \) and \( y \).

Based upon his observation of the angle measurements in the diagram, he concluded that \( \angle BDC \) and \( \angle BAC \) were supplementary. Thus, he was able to quickly obtain the solution.

The Solution: \( x = 38 \) and \( y = 76 \)

Cahill’s solution worked for the problem that he presented but his method did not work for the following problem. (WHY?)

Similar Problem

On this problem, using his method, Cahill’s solution would be \( x = 28.5 \) and \( y = 57 \).

Question

What relationship must exist between the given angles so that Cahill’s method of solution works?

Cahill’s Conjecture

The class formulated the following statement which was appropriately called Cahill’s Conjecture.

If two isosceles triangles share a common base, \( BC \), and the vertex angle of triangle \( ABC \) is twice one base and of triangle \( DBC \), then the vertex angle of triangle \( DBC \) is twice one base angle of triangle \( ABC \).

Generalization

Once stated, the class generalized and proved that Cahill’s conjecture was true for all cases.
\[ \triangle ABC: \ a + x + x = 180 \quad \Rightarrow \quad 2x = 180 - a \]
\[ \Rightarrow \quad x = 90 - \frac{1}{2}a \]

\[ \triangle DBC: \ \frac{1}{2}a + \frac{1}{2}a + y = 180 \quad \Rightarrow \quad a + y = 180 \]
\[ \Rightarrow \quad y = 180 - a \]

Therefore, \( y = 2x \).
Thus, Cahill’s Conjecture is true.

A student’s method of solution can enhance the learning of every student in class if we, the teachers, are willing to adjust our lesson plan and take the opportunity to investigate the solution with the class.

Try These

To conclude, try the following two problems with your classes. Have the students do the following:

1. Solve each problem.
2. Make a connection between the given number(s) and the solution.
3. Make a conjecture.
4. Write the conjecture in “if, then” form.
5. Prove or disprove the conjecture.

Problem 1:
\( ABCD \) is a square. Point \( M \) is the midpoint of \( CD \). \( AM \) and \( DB \) intersect at point \( Q \). \( QR \perp AD \). \( AB = 8 \). Determine \( QR \).

Problem 2:
\( a \parallel b \). Determine the value of \( x \).

Front Row (seated) L to R---Ryan Rumph, Alyssa Montalto, Sean Cahill, Leanna Anderson, Liz Awotwi
2nd Row (seated L to R---John Borsellino, Jackie Neustadt, Dalila Camacho, Megan Soger, Emily Kopija
3rd Row (standing) L to R---Jackie Glowinski, Mike Rakosnik, Devon Marino, Samantha Kanak, Melissa Farrell, Kara Schubert, Ashley Saviano
Back Row (standing) L to R---Patrick Baier, Dave Albaugh, Dave Lembas, Garrett Goebel, Dex Jones, Dan Dowjotas, Anthony Harding
Polygon and Plane Figures Terminology Activity

Denise Reid & Janice Lowe
dtreid@valdosta.edu
Valdosta, GA 31698

Included among NCTM’s five process standards of the Principals and Standards for School Mathematics is Communication. One goal of this standard is to “... use the language of mathematics to express mathematical ideas precisely” (NCTM, 2000, p. 60). In the middles grades increased attention should be given to mathematical terms and definitions (NCTM, 2000). The following classroom activities may be used to provide opportunity to study and learn terms about polynomials and plane figures.

The first puzzle is a find-a-word puzzle. It can be used for recognition of terms. It is a good idea to require your students to first find a definition of the terms before circling them in the puzzle. If their own textbook does not have a good glossary, your school library or the internet may be used.

Having first completed the find-a-word puzzle, the students should then complete the crossword puzzle. This task should be done after the find-a-word since many of the terms are repeated in this puzzle. We hope that your students will find both of these puzzles enjoyable and challenging.

References


The puzzles for this article are on the next three pages. The Solution for the Mathematical Find-A-Word is on page 30. The solution for the Crossword on Plane Figures is on page 48.
Mathematical Find-A-Word

All the mathematical terms from polygons and plane figures listed below are hidden in the array. Solve the puzzle by finding and circling them. The words are always in a straight line and may read up, down, left, right, or diagonally.

When you have found all of the hidden words, there will be some letters remaining. Read these from the top, left to right, and they will spell out a sentence involving plane figures. Have fun!!

M A R G O L E L L A R A P
D E R E U L S P R E D E A
I T C C C U O H C R N T R
A I L R I L O T O T R H A
M K I D Y M A H A A E E L
E C A G B N C G P L R X L
T R O U G E O E G N A A E
E N S L S N Z N O I U G L
R D E U E O A G A N Q O G
E R T O I M A E T R S N Y
I B A D S C O C T A G O N
O F U Y E L G N A I R T N
Q U A D R I L A T E R A L

Angle  Hexagon  Pentagon  Rhombus
Arc    Kite       Polygon  Square
Chord  Obtuse     Quadrilateral Trapezoid
Circle Octagon    Radius    Triangle
Decagon Parallel   Rectangle
Diameter Parallelogram

Hidden Phrase _________________________________________________________________
Crossword on Plane Figures
CLUES

ACROSS

2.  A polygon with 8 sides
6.  The number of interior angles that a triangle has
7.  A quadrilateral with 4 sides the same length and 4 angles congruent
10.  The number of interior angles that an octagon has
11.  A quadrilateral with exactly two pairs of opposite sides parallel
12.  A one-dimensional figure that consists of one endpoint A, one point B, all of the points between A and B and all the points for which B is between them and A.
14.  A polygon with 5 sides
15.  The set of all points on a plane at a certain distance from a certain point called the center
18.  A quadrilateral with exactly one pair of opposite sides parallel
19.  The total number of sides and interior angles in an octagon
20.  A quadrilateral with 2 pair of adjacent sides congruent
21.  A quadrilateral with opposite sides parallel and congruent and all the angles are right angles
22.  A quadrilateral with 2 pair of opposite sides parallel and all sides congruent
26.  A closed curve created from the union of line segments meeting only at endpoints
28.  A polygon with 10 sides
29.  A polygon with 7 sides
31.  A chord that passes through the center of a circle
33.  A type of angle whose measure is greater than 90 degrees and less than 180 degrees.

DOWN

1.  The set of points that lie inside a polygon
3.  The number of degrees in each interior angle of a square
4.  A word used to describe the opposite sides of a rectangle
5.  The number of sides that a dodecagon has
6.  A polygon with 3 sides
8.  A polygon with 4 sides
9.  The type of interior angles that a square has
13.  The union of two rays that have the same endpoints
16.  A polygon with 6 sides
17.  The number of interior angles that a hexagon has
18.  Prefix meaning three
23.  A polygon is often described in terms of how many of these that it has
24.  The segment whose endpoints are any point on a circle and its center
25.  A segment whose endpoints are on a circle
27.  The number of diagonals that a hexagon has
30.  Part of a circle
32.  The number of angles in a decagon
Introduction

Students are too often dependent on the calculator for simple computations. Proficiency in mental arithmetic is an essential tool inside and outside the classroom. I define mental arithmetic here both as an action, computations without any external assistance, and as a medium which students may move through in their attainment of number sense. There is no reason for a student to be using a calculator to add single or double digit numbers unless it is done for the purpose of introducing that student to the structures formed by the number themselves. Allowing students to use the calculator for mental arithmetic sends a message to parents and, possibly, to school administrators that teachers may be misusing the calculator or are unaware of its proper use. Teachers need to be more cognizant of how students use the calculator and for what purpose.

Arguments from advocates and adversaries of the calculator

Calculators have been used in our schools to various degrees for over three decades. Yet, there is still controversy about their place and usage in the classroom. Proponents of calculator use, claim that calculators can be used to support the teaching of arithmetic, in particular, estimation and the order of operations (Gowland, 1998). However, detractors have said that the use of calculators can obstruct the development of children’s mental and pencil-and-paper arithmetic. In particular, calculator use has been blamed for preventing students’ understandings of numerical structures, concepts such as ratio and proportion, and properties such as distribution. Gowland conjectured that some of the opponents of calculator use may be teachers and parents who had to perform endless calculations when they were in school. These teachers and parents now subject the new generation to the same endless calculations in hopes of eliciting a better conceptual understanding of numbers.

Ellington (2003) synthesized a meta-analysis covering the calculator’s influence on students’ performance in the areas of operational, computational, and conceptual skills, as well as general problem-solving skills. After adjusting for outliers in the data sets for computational and conceptual skills, Ellington found that there was no significant improvement on tests of mathematical achievement without calculators by students who used calculators during instruction as opposed to those who did not. Ellington further concluded “students received the most benefit when calculators had a pedagogical role in the classroom and were not just available for drill and practice or checking work” (p. 456).

Increase in positive attitude has also been alluded to as an effect of calculator use. In another meta-analysis of research on calculators in mathematics education, Hembree and Dessart (1992) concluded that “those students using calculators displayed a better attitude toward mathematics and an especially better self-concept in mathematics than students who had no formal contact with the devices” (p. 25). The caution here, of course, is in the use of calculators by
teachers as a means of alleviating the anxiety that many students face when studying mathematics.

Some argue that calculators are nothing more than an imposed obstacle into mathematical thinking. One of those obstacles is cost. This is not only an issue for individual students but also for schools. It is ironic that sometimes colleges with the highest proportion of working-class students become the most captivated by expensive new gadgetry for teaching mathematics (Koblitz, 1996). When I tell students that they are required to buy a calculator for my classes, students often complain that calculators are a waste of money due to their lack of use or improper use in the classroom. However, calculators and technology in general are often a justified expense by administrators when seen from the perspective of being at the cutting edge of educational reform.

Others have blamed calculators for the “destruction within half a generation of a hard-won, effective algebraic symbolism…and its replacement by slavish verbatim copies of what appears in calculator displays” (Mackey, 1999, p. 3). It is worth reminding ourselves that techniques found to be effective in an experimental program with an enthusiastic instructor will not necessarily work under less ideal conditions (Koblitz, 1996). Under those less ideal conditions, a classroom-learning laboratory can quickly degenerate into a didactical rule-base environment focused on button pushing. In this environment, students often rely on calculators to produce answers rather than for aiding conceptual development.

The importance of mental arithmetic

So we finally arrive at some pivotal questions. Is mental arithmetic the key to quieting the calculator debate? If mental arithmetic requires the ability to perceive and manipulate numerical structure, then which numerical structures are involved? How should we help students develop an understanding of these structures and an ability to exploit them for computational convenience?

Ralston (1999), who emphasized a combination of a mental arithmetic and a calculator curriculum without pencil-and-paper arithmetic, suggested that students learn to perform mental arithmetic as soon as any arithmetic idea beyond counting is introduced. Dessart et al. (1999) believed that students and teachers should be able to distinguish among mental arithmetic, pencil and paper, and calculators as tools of computation. They would “chastise any student who reaches for the calculator to find $3 \times 4$…suggest pencil and paper for calculating $27 \times 340$ and…insist on using the calculator for $2.7568 \times 345.8972$ after the student estimates mentally an answer of 900 ($3 \times 300$)” (p. 6). Ralston assured us that significant advantages can be obtained by learning to do multi-digit mental arithmetic aside from calculational efficiency. Most children will probably enhance their number sense and learn how to organize mentally a non-trivial thinking process. Furthermore, development of multi-digit mental arithmetic requires just the kind of mental preparation in logical thinking that mathematicians have always believed is an advantage of studying their discipline regardless of the subject matter learned.

Mental arithmetic also assists us in our daily lives. According to a National Council of Teachers of Mathematics (2005) position statement, “even more important than performing computational procedures or using calculators, students need greater facility with estimation and mental math than ever before” (p.1). Skills such as these are indispensable for understanding numbers.
and because of their usefulness outside of school (NCTM). After all, it is impractical and cumbersome to carry a calculator at all times in order to conduct routine business such as buying food and clothing, going to the movies, calculating a restaurant tip, or tipping a taxi driver. Unfortunately, in our classrooms today, we still see the calculator routinely used as a replacement for mental arithmetic.

A possible consensus on the appropriate use of the calculator

An emphasis on mental arithmetic in the curriculum would not only help our students in high school and college develop a stronger conceptual understanding of mathematics but also would invite the question of how to develop and implement the appropriate use of calculators in our classrooms in order to complement mental arithmetic. Children should not be using their calculators for performing simple or repeated calculations. From this they will gain very little. They should instead be allowed to use them for any calculations for which an adult would also require a calculator (Gowland, 1998). NCTM (2005) attempted to resolve the need for both computation and for calculators in the mathematics classroom. “Students need an understanding of number and operations, including the use of computational procedures, estimation, mental mathematics, and the appropriate use of the calculator” (p.1). In a survey about calculators for the May/June 1999 issue of Mathematics Education Dialogues, there were some interesting comments from educators. Most recommended that a revision of the mathematics that is taught is warranted to acknowledge the power of calculators but that calculator usage should appear only after teachers were knowledgeable about the equipment and students had received appropriate instruction. The majority also believed that calculators should be used only after students had attained the knowledge to do the relevant mathematics without them (Ballheim, 1999). So this leaves us with a motivation for educators to teach and be taught the appropriate method(s) needed to integrate the calculator alongside mental arithmetic.

Suggestions for improvement of mental arithmetic

So exactly how do we promote the use of and development of mental arithmetic? I can already hear teachers arguing that some students are more comfortable with the calculators because their abilities are too limited to successfully use mental arithmetic. Although there are some special needs students for whom this is the case, many lower-performing students have much untapped potential. I can only suggest that teachers take every opportunity to enhance those students’ skills at mental arithmetic and number sense with games or other activities. Do not let previous low performance be an excuse for calculator abuse. Gowland (1998) suggested that one way to enforce mental arithmetic is with games or activities. Three examples of such activities are provided below.

- Find the numbers 1 through 20 using only four 4’s and the basic operations of addition, subtraction, multiplication, and division (i.e. \(7 = 4 + 4 - \frac{4}{4}\)).
- Find the numbers 1 through 30 using the numbers 1, 2, 3, 4 and the four basic operations (i.e. \(6 = 2 \cdot 4 - 3 + 1\)).
- How many sums can you find with the answer of 4, 5, or 6 and so on.
Another proposition for enhancement involves deciding when to use calculators. Thompson and Sproule (2000) suggested a framework that lets teachers decide when a calculator is essential or nonessential. Essential activities would be too cumbersome for students to complete without using a calculator. Nonessential activities can be completed without the use of a calculator. These activities can be process orientated in order to promote students’ understanding of the processes associated with mathematical exploration and problem solving. They can also be product orientated, indicating that students are to determine a computational solution or end product. Examples of calculator-essential activities can range from investigating the decimal-expansion patterns of fractions with prime denominators and statistical analysis of real-world situations. Nonessential calculator activities can range from mental mathematical games and problems with probability, ratio, and proportions (Thompson & Sproule). Below I provide three examples that can be used in the exploration of when to use mental arithmetic or calculators.

- The sum of the first eighty positive odd integers subtracted from the sum of the first eighty positive even integers is what.
- What is the value of \( n \) in the following equation:
  \[
  3^{2002} - 3^{2001} + 3^{2000} - 3^{1999} = n(3^{1999})
  \]
- What is the value of
  \[
  10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1
  \]
  Another suggestion is to use mental mathematics as warm-up or closing activity in a secondary classroom setting. With five minutes at the end of a period, students can work quietly on a few problems presented on the blackboard, no calculators please, share answers with partners, and finally share and compare strategies as a class (Rubenstein, 2001).

Final Comments

Proficiency in mental arithmetic is an essential tool inside and outside the classroom. I am not arguing for the abolishment of the calculator or its continuous use but only for its appropriate use with mental arithmetic acting as a keystone. With proper use, calculators make it possible for students to attain new concepts that were previously too time-consuming to compute by hand. Students can now study regression, correlation, and modeling in high school mathematics. Students can also use matrices to manage information, mathematical situations, and solve systems of equations (Burrill, 1999). On the other hand, left as an instrument for the substitution of mental arithmetic, calculators only inhibit the development of number sense and logical thinking as well as the subtleties and use of estimation and computational procedures. “If students have never been asked to solve problems without calculators and if they have not learned calculator-free strategies, then mental math will never be an option they choose” (Rubenstein, 2001, p. 443). I hope that we can overcome our students’ over-reliance on calculators so that we no longer wonder what happened to allow such degradation of their mental abilities.
References


Analyzing Energy Savings: A Modeling Analysis

David R. Duncan and Bonnie H. Litwiller
David.Duncan@uni.edu
Cedar Falls, Iowa 50614-0506

Mathematics teachers are always looking for connections between mathematics and the real world. We shall illustrate a situation in which such a connection can be made.

Many homeowners are now purchasing or considering the purchase of an “energy efficient” natural gas furnace. In making such a decision one must choose among different levels of efficiency and prices.

Suppose that Beverly’s current old natural gas furnace needs replacing. Her old furnace is approximately 40% fuel efficient, that is, it burns productively 40% of the fuel that it uses. The remaining 60% is wasted. Her heating gas bill averages $1000 a year.

Beverly is now deciding between two new furnaces. Furnace #1 is 90% fuel efficient and costs $1800 while furnace #2 is 99% fuel efficient and costs $2700. Which is the better buy?

To analyze this problem, Beverly notes that only 40% of $1000 (or $400) of her heating gas bill is actually used to heat her house with her current furnace. The other $600 is wasted. How much would it cost to heat the house for a year with furnace #1 (90% fuel efficient)? Assuming 100% efficiency, it will cost $400 to heat the house, and we can use this to calculate the cost of heating using furnace #1. To do so solve the equation: 400 = .90x where x is the total fuel bill for furnace #1; x = $444.

Comparing this $404 bill to the projected bill of $444 for furnace #1, Beverly notes that there would be an additional savings of $40 per year if furnace #2 is purchased. Since furnace #2 costs $900 more than furnace #1, it would take 22.5 years to pay the difference. This comparison situation is depicted in Figure 2.

Comparing furnace #2 to the old furnace directly, Beverly would save $596 per year; this savings would pay for the new furnace in about 4.5 years.

Which furnace should Beverly buy? Most of the savings are achieved in going from the old furnace to furnace #1. The
additional savings achieved by going from furnace #1 to furnace #2 seem to be minimal.

Are there other factors that should be considered?

– Environmental factors

– Possible increases in the price of fuel (Redo this problem if the price of fuel doubles.)

– The life of a new furnace

The reader and his/her students are encouraged to locate and examine other such real world problem solving situations.

Mathematical Find-A-Word (Solution)

The remaining phrase is “Euclidean Geometry is fun.”
How do you rate as a teacher of problem solving?

Barbara O’Donnell
Edwardsville, IL 62026
bodonne@siue.edu

Are you an effective teacher of problem solving? Teachers often wonder if their students are really thinking critically and reasoning to solve difficult, demanding problem solving tasks. They wonder if they are doing the right things to make sure that problem solving is happening in their classrooms. During the past three years, I have watched teachers implement problem solving. After observing and researching their problem solving practices, I have developed a quiz that might help you decide if what you think you are doing in your classroom is really making students solve problems.

The Quiz

Please answer the following questions to the best of your ability:

1. Do you know how to choose good problems that make connections between mathematical concepts?
2. How do you handle the stress that students experience during problem solving?
3. Do you know each problem inside and out? Do you analyze each problem to tell what concepts it teaches and where students may get stuck?
4. Do you have a bank of thought-provoking questions prepared to ask to make students do the reasoning?
5. After introducing and discussing the problem solving task, do you hand over the work to student groups to solve the problem?
6. Do you let students determine the constraints of the problem?
7. When students ask you questions, do you ask them questions back to make them think?
8. Do you fall into the trap of explaining how students should attack the problem or giving too many “hints” that guide students too much?
9. When problem solving groups hit a snag, what do you do?
10. Are you prepared for groups that finish early?
11. When the session ends, do you make sure that all problem solving groups have the same correct answer?
12. Do you let students debrief and take the sideline to listen to their ideas?
13. Do you leave problems up in the air to motivate students to explore further?

The Answers

So how did you do? Let’s check. With the help of some educational researchers and the veteran facilitators of problem solving I observed, I propose the following answers:

1. Choosing a problem that makes connections between mathematical
concepts is difficult. A way to make sure that you are doing this is to compare your problems to the Task Analysis Guide found in *Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development* (Stein, Smith, Henningsen, & Silver, 2000, p. 16). Problem solving tasks are sorted into four categories: memorization, procedures without connections, procedures with connections, and doing math. Although students need to have practice with problems in all these categories, problem solving needs a higher level of cognitive demand and the problems you chose to use should be classified as procedures with connections or doing math. These tasks demand engagement with concepts that underlie rules and formulae, require complex non-algorithmic thinking, can be represented in multiple ways, and stimulate students to make purposeful connections between mathematical concepts, processes, and relationships (Ball, as cited in Stein, Smith, Henningsen & Silver, 2000). Compare the following sample problems from this text (pp.1-2):

<table>
<thead>
<tr>
<th>Task Type</th>
<th>Problem</th>
</tr>
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</table>
| **Low level task** (Procedures without Connections):**  
Martha’s Carpeting Task**      | Martha was recarpeting her bedroom, which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase? |  
| **High level task (Doing Math):**  
The Fencing Task**             | Ms. Brown’s class will raise rabbits for their science fair. They have 24 feet of fencing with which to build a rectangular pen.  
a) If Ms. Brown’s students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?  
b) How long would each of the sides of the pen be if they had only 16 feet of fencing?  
c) How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

If you are serious about choosing good problems, the best thing you can do is to read the text above. In addition to helping you understand how to analyze each type of task through examples and cases, it also illustrates how to maintain cognitive demand throughout a lesson.

2. Problem solving can really create stressful situations. Members of groups can get into heated discussions about what strategy to use, what the correct answer is, the validity of each others’ ideas, etc. Sixth grade teacher Brenda’s students were intrigued by problem solving, but were also afraid to take risks and possibly be wrong. One of her largest obstacles was overcoming her personal doubts about letting students struggle. She came to realize that struggling with problem solving resulted in more confident problem solvers. She worked to create an environment that fostered dialogue, struggling, risk-taking, critical thinking and sharing of ideas, not competition.

3. Do you need to know each problem inside and out? I used to think that I shouldn’t know every detail of a problem because I didn’t want to lead students to specific strategies that I might use to solve it. Now I know better. As I review specific problems that I use year after year, I realize that the more I know about the problem, the better I can relate the solving process to other related mathematical topics. An example of this can be illustrated by the Locker Problem and the work of another middle school teacher, Belinda. In the problem, 100 lockers are opened and closed based on a sequence of multiples. Students can make a chart or diagram or work through the problem with manipulatives. As they
do so they often jump to conclusions about the patterns they are recording. They see a pattern of open lockers that increases by two between each closed locker. They can then solve for the number of closed lockers under 100, but still may not realize why they are closed. Too late, Belinda realized that she did not understand each step of the problem and what students may be thinking, so the reasoning and discussion behind why some lockers remain opened and others closed was superficial. She needed to redirect their reasoning by asking questions like, “Why are these particular lockers closed?” “How many people touched the closed lockers?” “What makes the closed lockers different than the open ones?” “Is there a reason why some are touched an even number of times or an odd number of times?” and so on to get at why the answers make sense. Belinda’s students never understood the relationship between the number of factors of each number and if its corresponding locker was open or closed. So don’t be surprised if you need to think and research the problems and mathematical ideas behind them, just be sure you are not leading them to a solution.

4. Yes, you need to prepare questions to help students make the connections that are present in a higher level problem. What better time to prepare your questions, than when you are working through the problem yourself. When you find a problem that really challenges your students, save the questions you ask with your lesson plans. Each year that you use your old standby problems, you will see new ways to make math connections and add more questions to your list. Of course, there are questions that will work for almost any problem, as suggested by the Professional Teaching Standards for Teaching Mathematics (1991) and cited in Mewborn and Huberty’s article, Questioning Your Way to the Standards. Some of them are:

“Does anyone have the same answer but a different way to explain it?”
“Can you make a model to show that?” “What would happen if…?”
“Can you predict the next one?” “Can you think of a counterexample?” “What is alike and different about your method of solution and hers?” “What ideas that we have learned before were useful in solving this problem?”(p. 244).

5. Hopefully, you answered ‘no’ to this question. After years of teaching mathematics through problem solving, I came to realize that each student needs to think about the problem before being immersed into a group discussion. The strategy “Think, Pair, Share” comes to mind (Lyman, 1981). After the problem is presented and questions concerning the problem are discussed, students need time to formulate their own ideas and the strategies they might use to solve it. Brooks & Brooks (1999) believe that “immediate responses prevent students from thinking through issues and concepts thoroughly, forcing them… to become spectators as their quicker peers react” (p. 115). Allowing 2 to 5 minutes for individual thinking pays off during the problem solving process because students come armed with ideas to share. One teacher I observed (Carol Aljets, personal communication) not only requires that students think on their own first, but asks students to have ideas
written on paper before they share with a classmate.

6. What are the constraints of a problem? They are the little details of the problem that can skew the results of the possible answers that students get. For example, in a problem involving the strategy “make an organized list” students are asked to find all the possible license plates that can be made with the numbers 1, 2, 3, and the letters A, B, C. In the discussion of the problem, students need to analyze the wording to decide if letters and numbers can be used more than once, and if letters and numbers can be mixed. These limits can be decided by the students and will make a difference on their outcomes. If students do not understand the constraints of the problem, their answers will not converge on an appropriate answer.

7. Instead of guiding students to a solution through explaining how to do the work or even giving innocent “hints,” teachers need to encourage inquiry by asking open-ended, thought-provoking questions (Brooks & Brooks, 1999). One approach is to ask students to elaborate on their initial ideas. Brooks & Brooks believe that “Through elaboration, students often reconceptualize and assess their own errors” (p. 111).

What to say when asked a question is difficult to determine. Laurel, a fifth grade teacher, describes how she handled such a dilemma:

Every group wanted help, a clear direction to the solution, but I remained neutral. When I questioned or commented to a group, the other groups were ‘all ears,’ listening for information that they may find useful. Finally, they realized that I wasn’t going to lead them to an answer, and the class began to develop its own ideas. They stopped talking to me and either sat in pondering silence or actively discussed ideas in their group... Giving a guiding question and/or questions without giving the answer is hard.

When in doubt about how much to say, Laurel suggests that you ask students to tell you what they have tried so far and show their work. Ask questions like, “Explain your ideas behind using this strategy.” “Did you ask all of your group members this question first?” “Is there another way you can check to make sure you are on the right track?” Students often get frustrated when teachers take this approach, but that is okay. In Laurel’s case, she persisted and students began to think for themselves. The question is “Can you outlast them and their frustration to make this change in problem solving ownership?”

8. Instead of showing students methods to try, ask students to share some of their ideas for how they might do the problem. Brooks and Brooks (1999) state, “When teachers share their ideas and theories before students have an opportunity to develop their own, students’ questioning of their own theories is essentially eliminated” (p. 107). I have observed teachers following this lesson sequence: 1) present the problem, discuss what it means and its constraints, 2) students think individually and formulate a strategy, 3) students share strategies within their group, 4) groups think about each strategy and check out its feasibility, then 5) groups share what strategies they think might work best. This additional
discussion point helps groups who might be heading toward a dead end without giving them too much information. Groups have not yet arrived at a solution, so no answers will be shared. This additional step is especially helpful if the task is very challenging.

9. What to do when groups are stuck, and the problem seems to have no end in sight? Does the teacher take over and work it out? No. It is time for a whole group discussion. I have found through my own experiences and that of other teachers, to ask each group to report on the strategies they have tried, this gives other groups the opportunity to look at the problem from a different angle. After all discussion is exhausted, send groups back to work. Repeat the process if necessary or refer to the answer to number 13.

10. Problem solving is like a maze, each group begins moving through the maze at their own pace, making turns at random. There always seems to be one group that makes all the right turns at the right time, so the teachers I observed prepared an extension to the original problem. One easy way is to change the constraints determined by the students earlier and ask the group to report on how this change affects the answer. Another way is to modify the original problem with more variables or larger numbers to make it more difficult. In any event, you need to be prepared for those fast finishers.

11. When the problem solving session is over depends on you. Does every group need to have the correct answer? Any answer? Do half of the groups need to have the correct answer? Is the time for math over? The teachers I observed generally had a good idea of when to stop the session. They felt that at least half of the groups should have a reasonable answer and be working on the extension to the problem while other groups may have arrived at a conclusion that may be partially correct. In some instances, a group may not be close to an answer, but that is okay. Students are building experiences that will help them with future problems.

12. When the time comes to discuss the problem, possible strategies to solve it, connections between mathematical concepts, and the answer, students should be doing the talking (Brooks & Brooks, 1999). At this point in the process, they are the experts on their own work. Taking away the excitement of sharing their work is a crime. Mary Kay, a fifth grade teacher believes that her “main role is to study my students, assess their progress, and organize sharing time so students view a variety of solutions… to help students build concepts, learn new strategies and reflect on their own learning.” In this vein, hand groups overhead transparencies and markers and ask them to explain every step of the path they took to get to a solution. Should the teacher ever step in? Only as a facilitator! Ask groups to clarify their ideas, ask students to share their failures as well as the successful strategies they used, ask more questions to get the students to do more in-depth explaining (Brooks & Brooks, 1999).

13. Yes, give students time to mull over difficult problems. One class period is not always enough to solve a really difficult problem, present it, and discuss it. Leaving a problem ‘up in the air’ piques students’ interest. Laurel remembers the day when “…thirty
minutes passed, no groups had a sure method formulated to solve the problem.” Laurel maintained her position as facilitator, and these students continued working on the problem the following day until they figured out strategies leading to viable solutions.

Given these situations, students often go home talking about the problem to their families or other classmates. How often do students talk about math and show a real interest in it? Not often enough (Burns, 1994). This is a way to make those conversations happen.

How well did you do?

Did you agree with the answers? Remember, these answers are based on specific teachers’ experiences, and do not purport to be the definitive answers or to cover every element of problem solving scenarios. They will however give you a glimpse of what happens when teachers use effective strategies to help their students become critical thinkers and problem solvers.

In self-help tests such as the ones you see in magazines at checkout isles of grocery stores, a score would help you determine your aptitude based a range of correct answers. Instead of telling you that a specific score means you are a novice, an apprentice or an expert, this quiz is designed to make you think about your practice and compare it to these experienced teachers. So how do you think you did?

References


Old Ideas Revisited

Mary A. Thomas
mthomas@d211.org
Schaumburg, IL 60193

Tired of the same old same old? Attached are a series of old ideas revisited. All of these ideas have come from ICTM conferences and colleagues over the years. I have simple re-worked them to fit my needs. Everything we share has value, but sometimes a little modification of the activity can turn it into something that works for you. I hope you are able to “tweek” them to fit your needs. The bottom line for me is trying to make class fresh and interesting for our students along with teaching toward commonly valued course objectives. We do not have complete control over whether our students leave our room bored, but we can plan to shake things up and hopefully leave our students wondering where the time went instead of watching the clock. The old ideas that I will be revisiting include openers, people searches, stations, matching activities, little books, and exit slips. Many more exist. Have fun!

PSAE PowerPoint Openers or PODS

The attached openers target Prairie State Achievement Examination (PSAE) style questioning. Whether the questions are tossed on the overhead, the chalkboard, or projected from the computer, everyone teaching algebra in your building can share the same questions. Some teachers use PODS or problems of the day to accomplish the same thing. Appropriate test taking strategies can be discussed naturally without separating themselves from the unit at hand. Variations on this theme include having the students save their correct questions for points at the end of the term. Some teachers prefer to collect them back immediately. Others simple ask that this be the first things in the student’s notes everyday. Experiment and find what works for you and your class.

People Searches

A “people search” is an activity where students must search out other students to check their work. People searches are a method of completing practice problems that promotes student interaction and communication about mathematics. The attached people search is one example of an activity that is easy to replicate and re-use year after year. There are several ways to handle people searches.
and I enjoy shaking things up and trying different approaches. Sometimes, I ask the students to complete all the problems individually for 10-15 minutes and then circulate the room for 5 minutes to get different students to approve their work. The idea is students helping students with dialogue about mathematics. A timer or a time limit can generate a more focused approach. It also helps students see and value the contributions of their classmates. Another variation on this theme is asking the students to find a different female student for every even and a different male student for every odd. To decrease the quantity of grading, I frequently take the last five minutes of this activity and assign students to put the problems around the board for everyone to check. Pre-numbering the board provides a visually clear opportunity for all students to check their paper. Please see graphic 1 at the end of this article for a sample.

Stations

Another oldie but goodie is a stations activity. Stations are an interactive small group activity that can be used to break up the monotony of essential practice. I use stations for test preparation, general review, and to keep students focused prior to school breaks. There are various themes on this activity as well. When I use this activity as a review prior to a test, I prefer to provide the answers at the next station. I feel it promotes students evaluating their own strengths and weaknesses. Providing the answers makes it clear what specific areas the student needs to review prior to the test. The stations I use for review prior to the test could serve as a practice test if formatted differently. When I use the activity for general review, I usually assign each group a station to present on the chalkboard at the end of the activity. Again, pre-numbering the board tends to help me get the visual results I want. Depending on the time available, I frequently ask the students to present their solutions. I recommend role modeling the result you want in terms of presentation in the beginning and use follow up questions to highlight the key information that you want students to remember. I also enjoy using this activity the day before breaks. Allowing a little off-task conversation in a controlled way tends to work for my classes prior to big holidays. One variation on the theme includes posting the stations around the room on the wall with tape. When I find a class that can not yet quite handle the responsibility to physically rotate and stay on task, I have rotated the poster or slip of paper containing the station. I have witnessed teachers using more stations than six, but I prefer fewer stations and more time for presentation and discussion. I form the desks into six groups of four or five and signal the rotation with a quick flip of the light switch and a slightly annoying “Rooooooootate!” A simple collection of the papers once or twice gets the point across that it is not free time. Again, a timer can reduce the “how much time is left?” question, but for me I generally do not use a timer. Rather I go by observation. If everybody is still actively pursuing the questions, I can go a little longer. I think this comes with practice. Designing stations that take about the same time also helps.

<table>
<thead>
<tr>
<th>Station #1: Multiplying Polynomials</th>
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<tbody>
<tr>
<td>1) ((x+4)(x-4))</td>
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<tr>
<td>2) ((x-4)(x-4))</td>
</tr>
<tr>
<td>3) ((3x+2)(2x-1))</td>
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<tr>
<td>4) ((x+1)^2)</td>
</tr>
<tr>
<td>5) ((x-4)(x^2+x+4))</td>
</tr>
</tbody>
</table>
Super Match Activities

Another great class activity is matching. I have attached an example that involves first and second derivatives just to demonstrate the value of these activities with both higher and lower level mathematics. Creating a matching activity doesn’t have to involve the same preparation time as this particular activity. Cutting up a solution key and simply reordering it on another sheet of paper will do the trick for most worksheets. I bring in the class set of scissors and box of glue sticks and let the students do the activity. Some teachers like to cut the pieces out for the students, but I enjoy allowing the students ownership of the cutting and gluing. Please see graphic 2 at the end of this article for a sample.

Little Books

Little books can be made by folding two index cards together and stapling them in the middle. I use these little books to frame students summarizing massive amounts of information into key parts before major unit tests. For instance, in trigonometry, during the major unit on identities, I ask my students to prepare a “Little Book of Trig Identities.” In AP Calculus A, I ask my students to summarize the first four chapters of material in a “Calculus Digest.” It’s the same idea as asking students to whittle down the key information on an index card. The “little book” notion makes the creative student enjoy it more. In the same manner though, more analytical students sometimes find the task less enjoyable because of the creative bent. Balance is everything. For me, the results over the years have been amazing. Sometimes we include a tribute on the inside front cover just for fun. Other years, we include an author’s note on the back. This
past year, it was a Calculus Haiku. Rarely are these activities completed during class time. Instead they are given in advance of a major test or final to motivate review. All we do in class is set the framework and staple the note cards. On occasion, I notice that students forget very key information. This then serves as a notice to me to specifically mention this information again in class. For example, after the applications of derivative’s unit, I noticed that several students included nothing in their little book on Newton’s Method. I then intentionally reviewed Newton’s Method the day before the test.

Exit Slips

Exit slips help manage the last 1-3 minutes of class when needed. It is a quick way to summarize the lesson for the day or monitor quickly the progress of students with the main concepts. The exit slip can have various themes, including reflection, self-evaluation, summarization, and teacher feedback. Using two or three basic concept questions for the day’s lesson can also give quick feedback. These questions do not have to be graded, but rather serve as informal feedback to the teacher. Frequently, I am surprised by some of the honest feedback. Sometimes, all I’m trying to do is make the student reflect even if only briefly on their crucial role in the learning process. Some examples are included below.

Exit Slip
Rate how you feel about simplifying radicals from 1 to 10 with 1 representing “What are radicals?” and 10 representing “Give me the test now, because I’ll ace it!”

Exit Slip
Are you receiving enough examples in the notes to make the homework assignment reasonable? Explain what you think needs more coverage.

Exit Slip
What do you need to do to be prepared for the test?

Exit Slip
1) Rate the quality of your notes from 1-100.
2) Can you find Feb. 2nd notes?
3) What was the third example on Feb. 5th?
4) Who do you consider the most reliable student to get notes from after an absence in class? Why? Please answer in complete sentences.

Conclusion

In conclusion, I hope this review of activities all originating from ICTM conferences and colleagues over the years will serve as a vehicle to spark your student’s motivation. Sometimes, a slight shift in a lesson plan can be all it takes to make class just a little more fun for all students, or invigorating to a student that may be working too many hours outside of school. Feel free to “tweek” away any of these activities to fit both your and your student’s needs. Ours is a great profession and no matter the challenges we or our student’s face, we must demonstrate the excitement of mathematics in a manner that fits the modern student. Combined with meaningful rigorous direct instruction and realistic examples, varying your mode of instruction to include a variety of intelligences can possibly make the difference between reaching all students or not. Have fun!
### Solving Equations with Radicals

#### People Search

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<tbody>
<tr>
<td><strong>1.</strong> Solve $\sqrt{x} - 7 = -5$</td>
<td><strong>2.</strong> Solve $\sqrt{x} + 5 = 4$</td>
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<tr>
<td><strong>3.</strong> Solve $\sqrt[3]{x} - 8 = -5$</td>
<td><strong>4.</strong> Solve $\sqrt[3]{2x + 1} - 3 = 2$</td>
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<tr>
<td><strong>5.</strong> Solve $\sqrt[3]{x} + 4 = 3$</td>
<td><strong>6.</strong> Solve $\sqrt[3]{x} - 4 = \sqrt[3]{2 - x}$</td>
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<tr>
<td><strong>7. Solve</strong> $\sqrt{x} + 7 = 12$</td>
<td><strong>8. Solve</strong> $\sqrt[3]{x} - 18 = -15$</td>
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<tr>
<td><strong>9. Solve</strong> $\sqrt{2x + 4} = 4$</td>
<td><strong>10. Solve</strong> $\sqrt[3]{x - 4} = 2$</td>
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<tr>
<td><strong>11. Solve</strong> $\sqrt{3x} + 5 = 0$</td>
<td><strong>12. Solve</strong> $\sqrt{x + 1} - 1 = 2$</td>
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<td>Answer ________________________</td>
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Graphic 2: Super Match Sample

**Super Match**

Name __________________

1) Cut out all first and second derivative
2) Paste them next to their original function

<table>
<thead>
<tr>
<th>F(x)</th>
<th>F'(x)</th>
<th>F''(x)</th>
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<td><img src="image17.png" alt="Graph" /></td>
<td><img src="image18.png" alt="Graph" /></td>
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</tbody>
</table>
Writing Across the Curriculum

Explain how you determine the correct first and second derivatives.
You must use complete sentences. Try to be concise but thorough.
First Derivative
Crossword on Plane Figures (Solution)
ICTM MEMBERSHIP APPLICATION FORM

Clip out this page and mail it with your payment to the address below.

☐ New Member         ☐ Reinstatement  ☐ Renewal         ☐ Change of Address

Name

Check preferred mailing address. Please complete BOTH columns.

☐ Home
   Street Address: _____________________________
   City: _____________________________
   State: _____________________________
   Zip Code: _____________________________
   Phone: _____________________________
   Email: _____________________________

☐ Work
   School Address: _____________________________
   City: _____________________________
   State: _____________________________
   Zip Code: _____________________________
   Phone: _____________________________
   Email: _____________________________

Regional Office of Education: _____________________________

NCTM Member? ☐ Yes ☐ No

Profession: (check only one)
☐ EC-3 Teacher
☐ 4-6 Teacher
☐ Jr. High/Middle Teacher
☐ Sr. High Teacher
☐ Special Education Teacher
☐ Community College
☐ College/University
☐ Administration
☐ Retired
☐ Student
☐ Institutional Member
☐ Other
☐ Other

Interests: (check up to three)
☐ Remedial
☐ Gifted
☐ Teacher Education
☐ Assessment
☐ Certification
☐ Multicultural Education
☐ Teacher Evaluation
☐ Professional Development
☐ Scholarship
☐ Technology
☐ Research
☐ Math Contest

Dues for ICTM Membership:

☐ Regular Member
   ☐ one year $25
   ☐ two years $45
   ☐ five years $100

☐ Retired Member
   ☐ one year $20
   ☐ five years $80

☐ Student Member
   ☐ one year $15

☐ Institutional Member
   ☐ one year $70

(The name of classroom teacher in the blank at the top of the page will be used as the contact teacher for the institutional membership. Please make sure to indicate a contact person.)

If recruited as a new member by a current member, please list the recruiter’s name

Mail this application and a check or money order payable to:  EASTERN ILLINOIS UNIVERSITY

Send application and a check to:
ICTM Membership
School of Continuing Education
Eastern Illinois University
600 Lincoln Avenue
Charleston, IL  61920-3099

Total Enclosed: $ _____________________________