THE ILLINOIS MATHEMATICS TEACHER

THE ILLINOIS MATHEMATICS TEACHER

Volume 54, No. 1 Fall, 2003

Table of Contents

From the Editors…

Welcome to another school year and the Fall issue of the *Illinois Mathematics Teacher*. It has been awhile since the *IMT* was last published, due to the shortage of submissions we have received. To keep publishing quality journals on a timely basis we need your help. Please consider submitting an article or a favorite classroom activity with an explanation to be included in future issues.

This issue of the *IMT* contains many useful and interesting articles and activities covering a wide range of topics. Measure for Measure introduces a rhyme to help young children with informal units of measure. Technology is covered in articles by Keary Howard, Melissa Scranton, Dianna Galante, and Patricia Nugent. There is a follow-up article on Lesson Study. Teaching with Cross Sums and Calculating Candy Combinations: A Puzzle Problem are both activities that can be used in the classroom. This issue introduces a new column, Teaching Math Through Just Doing It. Finally, David Peabody has contributed another poem, this time about hexagons and pentagons.

We would appreciate hearing from any of you out there who are reading the journal. We would especially like to hear about any activities from the *IMT* that you have used with your students. Your comments and constructive criticism are heartily solicited.

Thank you for sharing.

Marilyn and Tammy editors

Guidelines for Submitting Manuscripts

Prospective authors should send:

- **Five (5) copies of your article**, typed, double-spaced, 1-inch margins. Put your name and address only in the cover letter. No identifying information should be contained in copies of the manuscript. Articles should be no more than ten pages in length, including any graphics or supplementary materials.
- **A diskette with your article, including any graphics.** We prefer that the article be written in Microsoft Word and that it be saved on an IBM-compatible disk. Graphics should be computer-generated or drawn in black ink and fit on an $8\frac{1}{2}'' \times 11''$ page.
- **Your name, address, phone, and e-mail** (if available) should be included in a cover letter.
- **A photo of yourself (Illinois authors only),** color or black/white.
- **Articles may be submitted electronically to tvoepel@siue.edu.**

To: Marilyn Hasty and Tammy Voepel
Mathematics Department
Box 1653
Southern Illinois University Edwardsville
Edwardsville, IL 62026-1653

Measure for Measure

Mary Margaret Capraro
Texas A & M University
College Station, TX 77845

How many days old will you be on your next birthday? How wide is your hamster's cage? How tall is the tallest girl in your class? How long is it from the tip of your bird's head to the end of his tail? How many cups of lemonade can you make from the pitcher? How long was it from the time you got up in the morning to the time you went to bed yesterday? The only way students can answer these questions is to use measurement.

Thousands of years ago, the Egyptians started measuring things by using parts of their body as measuring tools. The Romans used the king's foot as a standard unit of measure. Since the king was unable to go from place to place and house to house to measure things, he decided he would be the ruler for measuring as well as for ruling his country and ever since King Henry the First, standard measuring sticks were used based on his foot.

According to the NCTM Principles and Standards for School Mathematics (2000) which has just included measurement as one of the five content strands, "measurement is the assignment of a numerical value to an attribute of an object"(NCTM, 2000 p.44). In order to measure something one must first decide on the attribute, such as length or area, to be measured. Then a unit is selected that has that attribute, such as inch for length, and lastly a numerical value is assigned by comparing the units by matching, covering, or filling the object to be measured.

Measurement can help us find out about people, animals, objects, and places. It also allows us to find distance, length, width, height, mass, weight, perimeter, area, capacity, and time using inches, feet, yards, miles, pounds, cups, pints, quarts, centimeters, meters, kilometer, grams, kilograms, liters, seconds, minutes, hours, days, weeks, months, or years. As a first step in the early grades, students begin measurement by comparing objects. These activities must be tactile. As students play with different objects, they begin to ask and answer questions such as is it longer or shorter, taller or wider, heavier or lighter. With further exploration as a natural progression, students order objects from hottest to coldest and fastest to slowest. Both in the classroom and at home, students need to explore varied experiences. Students should be given or allowed to use nonstandard or informal units of measurement such as sticks, ropes, straws, shoes, or hands and feet. Opportunities should be provided for students to use the nonstandard units of their choice, and then discourse should follow on how appropriate the measurement was and whether one student's choice was more appropriate than another's. With experience, students begin to use toothpicks to measure the length of their blocks and string to measure the circumference of their balls and not visa versa.

Until age seven, Piaget, Inhelder, and Szemininska (1960) and Paley (1981) found that children had no use for sticks, rulers, or paper. Carpenter (1972) and Lamb (1975) suggest that students about seven or eight can use numerical information in measurement situations. Hence, numerical clues can aid the child in logical reasoning tasks but only if they are at an age where they are most likely to have a true

understanding of numbers. These studies reiterate the need to meet children "where they are" and build on that knowledge to develop further understandings of measurement. In contrast, Clements (1999) and Steffe (1991) found that students can make their own rulers whether physical or "mental" and use them meaningfully to develop an understanding of length before they actually "conserve length" as suggested by Piaget.

Thus, there are no published guidelines about when to use standard and informal units of measure. As previously stated, students should begin with informal units and move to standard units and tools. The time spent on using informal units varies with the age of the child and the attributes being measured.

One thing is certain, students need many early experiences connected with measurement in real-world situations. The rhyme, *Hands* (1984) in Appendix A is just one example. Students in second grade measured their classroom door (see picture in Appendix B) and then went home with the rhyme laminated on construction paper with the task of measuring their dining room table's length in the same way. Before leaving for home, the class predicted whether all the students would get the same answers on their sheets. When the students came back the next day with the amount of hand spans it took to measure the length of their dining room tables, we checked their predictions. We then discussed the reasons why students had different measurements and whether all of their measurements could be correct. This discourse is necessary so students can verbalize their experiences and come to an understanding of what measurement is all about.

Finally, practice must be provided in distinguishing between the two processes of understanding the meaning and technique of measuring a particular attribute and learning about the standard units connected with measuring that attribute. Measure for measure, it is up to us as teachers to provide our students with interesting and exciting measurement activities so that they can grow and learn on their measurement journey.

References

Carpenter, T. P. (April, 1972). *The relations between the development of certain conservation and measurement concepts.* Paper presented at the annual meeting of the American Research Association, Chicago, IL

Clements, D. H. (1999) Teaching length measurement: research challenges. *School Science and Mathematics 99*(1), 5-11.

Davidson, A. (1984). 'Hands', 'Shapes' in *Ten little goblins*__(Understanding Mathematics), Shortland.

Lamb, C. E. (1975) *The effect of measurement on the child's transitive and substitutive reasoning in number and length contexts*. Unpublished doctoral dissertation, University of Georgia.

Paley, V.G. (1981). *Wally's stories*. Cambridge, MA: Harvard University Press.

Piaget J. , Inhelder, B., & Szemininska, A. (1960). *The child's conception of geometry.* London: Routledge and Kegan Paul. (Original work published 1948)

Steffe, L.P. (1991). Operations that generate quantity. *Learning and Individual Differences 3*, 61-82.

(2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics

Appendix A

Hands

One hand, two hands, three hands, four!

Four hands wide is the width of the door.

One, two, three, four, five, six, seven!

Seven hands tall is the height of Kevin.

Two hands, four hands, six hands, eight!

Eight hands long is the length of Kate.

Use your hands I'm sure you're able,

And find out the height, length, width
of a table.

Appendix B

Meeting the Challenge of Feeder District Articulation

John Dossey
Illinois State University
Normal, IL

Marion Hoyda
Lockport Township High School District
Lockport, IL

Many of the major problems associated with developing and providing a quality high school mathematics curriculum in a 9-12 school district relate to articulating the program with that found in the K-8 feeder districts. As the number of feeder districts grows, these articulation problems increase exponentially. The articulation challenges are not limited to curriculum. They also include the range in instructional patterns, expectations of student work, the use of instructional technology, and differences in students' opportunity to learn in the many classes of the various feeder districts.

Lockport Township High School (LTHS), District 205, located in Will County, is making headway in developing and sustaining a way of dealing with these challenges. LTHS has 7 public and 2 parochial feeder districts. As the Mathematics Department at LTHS began a study of its curriculum in 1999-2000, it noted that it would be difficult to dramatically impact student learning by limiting their content considerations to the curriculum at their level alone. Any hope of effective change for LTHS students required a K-12 study involving input from key teachers and administrators in all of the districts feeding the high school. This lead to the development of the Lockport Mathematics Consortium, a group consisting of such individuals. The participating districts of the Consortium fund the activities on a pro-rated basis so the expenses are shared and affordable.

In 1999-2000, the main activities of the teacher representatives in the Consortium were to examine the ISBE Student Learning Outcomes and to discuss how teachers dealt with them in the varied districts' classrooms. However, at the same time, the high school mathematics department engaged in planning that would eliminate lower level math classes so that all students, including significantly more students in special education, would start with Algebra I at the very least. This change added a sense of urgency to making sure all students were ready to enter LTHS prepared for Algebra I to be completed in one or two years. This vision, fueled by issues of equity and opportunity, served as the vision for the goals in the years that followed.

In 2001-2002 the work of the Consortium dealt with the development of a comprehensive curriculum plan focusing on the five state goal areas and establishing time guidelines for achieving the goal related outcomes within each of the grades from K through 8. Teacher representatives from every grade level, in every school, in every district, met to discuss the content and processes related to the state math goals. Consultants, John Dossey and Carol Thornton, from Illinois State University, led the professional development activities. Lockport Township High School administrators, Al Franz, Mathematics Chair, and Marion Hoyda, Assistant Superintendent of Curriculum and Instruction served as liaisons to feeder school superintendents and administrators to

coordinate the activities. Consultants and administrators persistently communicated the need for improving mathematics teaching and learning for the benefit of students. There is an important lesson here; the effectiveness of external consultants is buttressed through internal support.

The 2002-2003 activities focused on the professional development of a core group of teachers to deliver this curriculum in the Consortium's classrooms. However, the numbers of teachers involved in the instructional component of the professional development tripled from the previous year. These teachers involved serve, in a multiplicative sense, as building consultants relative to the common curriculum and some of the issues surrounding it. During summer of 2003, the Consortium will work to develop sample assessments for the various grades reflective of the curriculum and its stated outcomes. Teachers will design math assessments for grades K-8. The high school teachers will have a role in the design of the Algebra I assessments.

Why *A Common* Curriculum?

Before we examine the nature of the common curriculum, it is appropriate to examine why the decision was made to develop *a common* curriculum for the feeder districts. The first answer to this question is so that each district had a published and acknowledged curriculum. Several of the districts had an adopted textbook series, which served as a curriculum guide, but there were neither shared curricular goals nor any common grade level or within-grade expectations for the districts. While the Consortium districts had curricular guides, there was great variance in what actually took place in the classrooms, even within the individual feeder districts. None of the districts had an accountability program beyond standardized tests for their mathematics program. Hence, the development of a common curriculum was a start toward bringing some cohesion to the individual programs as part of strengthening the mathematics programs in the Consortium's schools.

A second reason for developing a set of curricular guidelines for the districts was the development of smooth sequences of courses to provide articulation in expectations and readiness on the part of the high school's program and for the students matriculating from the feeder districts. As the high school upgraded its program of studies to expect all students to have a significant algebra experience in or prior to the 9th grade, it was incumbent on the high school program to communicate these intentions to the administrators and faculty of the feeder districts. As more and more mathematics is expected of students at the high school level in order to be ready for opportunities and challenges at the collegiate, vocational, or career levels, the high school was moving to considering Algebra II a part of the core curriculum for all students. In addition, it was eliminating any pre-algebra courses from its curriculum. The high school will have a two-year sequence covering Algebra I and some geometry as the minimal level taught at LTHS. Algebra I, the year-long course remains in the curriculum. In fact, the changes taking place in the high school curriculum to assure that all students would have an algebra and geometry experience prior to graduation created a sense of urgency for all of the feeder districts participating.

A third reason for articulation was to strengthen and expand the current 8th grade algebra programs provided by some of the districts. Algebra I was also provided as a 0-hour class by the mathematics department at LTHS for students from the smaller feeder districts which could not afford to staff the class offering. While this 8th grade

algebra program has existed for 12 years as an option for the feeder districts, the high school wanted to expand the offerings for students in mathematics to involve more students in its upper division mathematics classes. The district and school administrators and staff have made persuasive arguments to parents and students for advanced mathematics study. As students begin to think of Algebra II as a part of the core curriculum, the mathematics department is working to increase the numbers of students staying in mathematics through the levels of AP statistics or calculus.

To accomplish such goals, the Consortium needed to adjust their programs so that more students could complete their study of Algebra I at the 8th grade level. The high school revised its identification matrix to allow greater numbers of students to enroll in Algebra I as 8th graders. By changing the matrix over two school years, the percent of students participating in the 8th grade algebra program will double, from 20 percent to approximately 40 percent by the fall of the 2003-04 school year. Initial results about increased numbers and student performance are very promising, which furthers the belief that more students are capable of learning algebra in the 8th grade than previously thought. (A long-term goal is to have 75 percent of students enrolling in algebra at the 8th grade).

For students not possessing the skills needed to meet the matrix requirements, another strategy is in place; transition math for 7th grade. Doing this in a safe, sane, and supportive way requires that participating students be identified at the sixth grade level and provided with a transition course at the 7th grade level. Such a course should provide coverage of the important topics contained in the traditional 7th and 8th grade curriculum. Alternatively, students could be identified in 5th grade and spend two such years in transition from arithmetic to algebra in grades 6 and 7. The transition course may actually be a separate class within some of the schools. It may also be completed in a summer program that builds upon the 7th grade curriculum taught during the regular school year. Making such changes requires consortium articulation of two types.

Articulation Considerations

The first type of articulation is that existing between grades within the feeder districts as students move from 5th grade forward. This means that they come to the 5th grade ready to start moving forward in fractions and decimals with a solid command of their work with whole number operations and the related facts. Secondly, it requires articulation with the high school program in assuring that the programs in algebra offered off-site from the high school are equivalent in coverage and expectations to that offered at the high school. This is important for two reasons. First, students need to be prepared both emotionally and content-wise to move on in mathematics in a seamless way as they make their shift to the high school. Secondly, they need to be assured that their work in algebra is accepted as meeting the prerequisites for the high school geometry course and other courses following the first course in algebra.

Another form of articulation is the development of student proficiency with the use of calculators and computers as they aid in the exploration of and representation of core mathematical concepts. Where appropriate, students need to begin to use the same forms of technology in the middle school programs as they will have available to them in their study of high school mathematics. As students enter their study of algebra, it is important that they become conversant and flexible in using graphing technology to explore relationships between algebraic expressions and their graphical

representation. Such knowledge includes an understanding of the relationship between parameters in expressions and rates of change, or slopes, associated with various lines; the recognition of when data is linear or non-linear; and a clear understanding of what the solution or root of an equation is and what the graph of a function might look like. Students also need to know both how to and when—and when not—to use a calculator, when to approach problems mentally or use estimating techniques, and when to intensively employ technology. Students with such knowledge are empowered to attack quantitative situations successfully.

Such teacher knowledge of curriculum and technology does not happen in a vacuum, nor does it come through reflective self-study. Neither does a solid understanding of the conceptual sequences that govern deep understanding of the subject matter material. Quality teaching of the material occurs when teachers know how students come to understand the material, the sequence of concepts and procedures that are required to move to the next level, and what prevalent misconceptions students may form. Two other aspects of teacher knowledge that are required are the ability to model critical ideas with manipulative materials or representations and the ability to show students how the material at hand is important in real–world applications. For this level of knowledge to develop, teachers need opportunities to have a concentrated study of the algebra curriculum, from the beginning stages in the primary grades through the applications of the algebra encountered in Algebra I.

Further, for change to really take place, new modes of structuring instruction for learning in an algebra classroom needed to take place. Classes totally dependent on lecture and paper-and-pencil work are no longer the accepted norm for Algebra I. Students who learn algebra in an empowering way need opportunities to examine situations involving change, growth, and proportionality in a variety of settings– investigating, experimenting, and learning to represent the relationships they see. These non-teacher centered learning activities do not indicate that the teachers are "not teaching" their classes. Changes in instructional methods call on teachers to become educational leaders of their students–often leading by engaging the students in material that is curricular and developmentally appropriate and challenging them to resolve the situations through employing what they know to develop new knowledge and understanding. Teachers in such classrooms employ a balance of pedagogical methods, ranging from small group work, to laboratory experiences with technology (calculators or spreadsheets) and manipulative materials (algebra tiles, measurement activities…), and whole class settings to provide overviews, to consolidate findings, and set the course for new materials. Such a repertoire of methods also requires the development of new skills. Finally there is the need for the development of new methods of assessment and evaluation to go along with the content and instructional approaches mentioned. The Lockport Consortium had developed a multi-year model to scaffold teacher, curricular, and assessment development to meet the requisites previously identified.

Meeting the Consortium Needs

In order to address some of the articulation issues, the Lockport Mathematics Consortium instituted a series of professional development sessions during the 2001-2002 school year. These sessions were directed by Drs. Carol Thornton and John Dossey of the Illinois State University Mathematics Department and held in the

professional development workshop room at Lockport Township's East Campus. Carol Thornton conducted the sessions for teachers of grades K-4 and John Dossey conducted those for teachers of grades 5-12 (a group of teachers from LTHS participated in each of the sessions to further develop articulation and lines of communication, as well as understand the curriculum that their students had encountered prior to coming to LTHS). The primary teachers were further divided into groups for K-2 and 3-4 and met in half-day sessions. The 5-12 teachers met in full-day sessions.

During the 2001-2002 year there were 7 professional development sessions for each of the groups of teachers. These sessions focused on the following topics:

- Changing Mathematics Education: Instruction, Assessment, & Problem Solving
- Number Sense and Number Operations
- Geometry and Making Connections
- Measurement Concepts and Skills and Reasoning
- Data Analysis/Chance and Communication
- Algebraic Thinking and Representations
- Putting It All Together – Building a Common Curriculum

Each of the sessions involved the teachers in a consideration of the NCTM and ISBE recommendations for curriculum within the content and process areas mentioned. Along with this, the teachers considered the major sequences of concepts, procedures, and processes taking place at the various grade levels and how these build on one another. Further, they examined the models and representations that might be used in helping students come to develop the understanding desired and meet the objectives set for each of the goals at each grade level.

The consideration of the national and state recommendations for student learning occupied a good portion of the time during the first year. Each of the grade level groups discussed what they did with the content related to these recommendations at their grade levels. They compared and contrasted their work to the recommended sequences and the ways in which their students learn. Time was spent working to align the instruction in the individual districts and match it up with the developmental and curricular sequences examined in the other portions of the workshop. Teachers also discussed their fears and concerns of delivering upon the expectations for the new curriculum with the consultants and each other. Slowly but steadily confidence replaced doubts. There may be a few teachers reticent to accept the changes, however, the change process continues with support.

By the end of the 2001-2002 sequence of sessions, the teachers participating in the Consortium workshops had developed a set of curricular outcomes for grades K-8 and had even made suggestions for focal emphases within the program at each grade level. These emphases were stressed limiting the number of outcomes per year in order to allow for greater emphasis on these specific topics within a given year. This was done with the goal of eliminating cycles of reteaching by teaching to a greater degree of mastery when the topic was considered the first time. In addition, a pacing guide was developed suggesting what topics should receive emphasis in which quarter(s) of the school year. The pacing recommendations help teachers maintain a focus on an entire year and establish a reasonable pace to assure that the topics critical to between-grade articulation received appropriate attention. These outcomes were shared at the building level with other teachers by those teachers

participating in the workshops. These outcomes were also discussed and reviewed with the district superintendents, curriculum specialists, and principals at a meeting in June, 2002. They were also distributed to all teachers and in some districts, officially adopted by the Boards of Education.

During the 2002-2003 school year, teachers participated in a second series of workshops conducted by Carol Thornton and John Dossey. These workshops focused on two different goals. The grade levels were split differently to provide grades K-5 a focus on number through decimals and grades 6-9 with a focus on algebra. This resulted in Grade 5 teachers shifting to work with the K-4 teachers. The K-5 sessions during 2002-2003 also involved a larger sample of the teachers from grades K-5 in the Consortium schools, many of which had not participated during the first year. Carol Thornton modeled instructional techniques, using a variety of manipulatives and mental processes aimed to increase student understanding. In addition, districts' special education teachers, including those from the high school, also joined in the workshops at the 6-9 level.

The 6-9 teachers' program in 2002-2003 focused more directly on preparing students for the algebra curriculum with the expressed purpose of developing a smooth transition to Algebra I in Grade 8 for those students for which such a program was appropriate. As a result, the teachers participating in the 6-9 sessions were essentially the same as the first year with some additions. The teachers at these grades were, in addition, more departmentalized in the schools across the feeder districts.

At the end to the 2002-2003 school year, the program of workshops had resulted in over 120 K-5 teachers in the feeder districts having completed either a one or two-year sequence of professional development in mathematics focusing on their curricula and instruction. Further, approximately 35 Grade 6-9 teachers had completed a second year of study of the mathematics curriculum at their level. Beyond the professional development, the teachers and districts had developed a common K-8 sequence of expectations for their curricula in mathematics. They also developed a special transition course in mathematics for use at Grade 7 for those students transitioning from Grade 6 mathematics in Grade 6 to Algebra I in Grade 8. The high school teachers of mathematics for special education students developed core objectives they shared with special education teachers from the middle schools as another means of articulation. This further communicated the high school's expectations.

As the teachers and districts look forward to 2003 and beyond, they are involved in developing common assessments and programs for technology usage within the districts that will bring them into further congruence. These steps will continue to strengthen their mathematics programs. The foci for 2003 and beyond include instruction, technology, assessment, and continued work on delivering the best curriculum possible to their students. The progress of this initiative continues due to the commitment at the district level and building level within the consortium. A long-term relationship with the consultants has accelerated the success and contributed to teacher confidence in the program expectations for themselves and their students.

Concentrated articulation over multiple years on a specific subject increases the likelihood of greater success for teacher learning and student learning. District and school leaders must "stay the course" and identify outcomes at the onset so they can benchmark their progress. They must also revise outcomes as they collaborate with

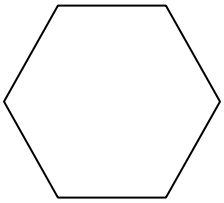
each other and the consultants and prepare for the logistics of communicating and planning because it takes centralized leadership to attend to the details. In the case of Lockport Consortium, the high school assumed those responsibilities. Lastly, accept the slow but deliberate start and enjoy the progress from stage to stage. The curriculum, instruction, achievement and use of technology will accommodate the new learning for the benefit of students.

If There Were No Pentagons or Hexagons

David Peabody

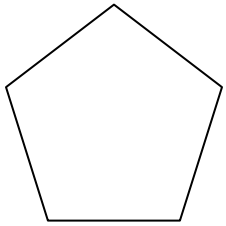
Let's pay tribute to two shapes for being alive
The Hexagon with six sides, and the Pentagon with five.

The Hexagon can exist in things inorganic,
Like nuts, bolts, and wrenches used by a mechanic.



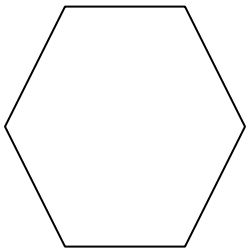
The bees that make honey in their combs made of wax,
Make Hexagon cells and use space to the max.

And that's also why Hexagons make good picnic bench seating,
If you build with this shape, more people are eating.



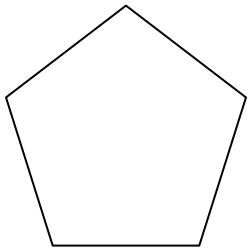
And in shapes of some crystals we hear the hexagon's voice,
Build with my unique shape and you've made a wise choice.

And now for the Pentagon with all of its strength,
A regular figure with five sides the same length.



If we didn't have Pentagons, much of life would be bare,
They're not as round as a circle, but more round than a square.

In natural forms this shape shows its powers,
In arms of the starfish, and the petals of flowers.



Cut the top off an apple and inspect the insides,
In the seeds of this fruit, a Pentagon resides.

So to have blueberry tops, soccer balls, and all flakes of snow,
These two beautiful polygons we just can't bear to see go.

31 July 2003

Joey Chitwood's Thrill Show Revisited: A Modeling Project/Competition Incorporating Graphing Calculator Technology and Mathematics in Grades 10-12

Keary Howard
SUNY Fredonia
Fredonia, NY 14063

Introduction

Over the last three decades, the Joey Chitwood Thrill Show has been a staple form of entertainment at county fairs throughout the United States. It's full of thrills, spills, and 'controlled' car wrecks. This project couples graphing calculator technology with Hot Wheels© cars to produce a modeling project/competition that fosters problem solving and fun for students in grades 10-12.

The project's genesis is traced to a mathematics and computer science challenge held annually at a local college where local schools assemble teams to participate in various cooperative problem solving activities. Individual teachers have since incorporated the project as a component of their pre-calculus curriculum, particularly while introducing students to modeling and regression capabilities with the TI83 calculator. The project effectively combines characteristics of graphing calculator laboratory activities (Bowman, Koirala, and Edmond, 2000, Lapp, 1999, Mingus and Grassl, 1997) with mathematics/engineering contests (Koes and Saab, 1996, NSTA, 1990, Kahanec, 1985) to produce a meaningful application of mathematics that can be readily incorporated into your curriculum. A complete description of the project is outlined, including instructional tips and strategies for teachers wishing to implement the activity.

Project Description

Each team (typically two to four members) is given an identical car, track, and ramp platform. Alternatively, you can purchase a single Hot Wheels track and a few identical cars for each team and provide times for selected practice sessions. Awards are typically offered in two events: (1) single jump distance and (2) group presentation of the appropriate mathematical models and equations.

The measure of success in the first event is extremely simple. The winning team jumps their vehicle the farthest, given only gravity and the materials provided. A single jump is scored for each team. Prior to the scored jump, the following data are recorded and verified for each team:

Constants

- The height of the release (launching pad) is 30 inches, the height of a standard table.
- The car must travel along the entire length of the three pieces of track provided (60 inches).
- A ramp of 4.5" x 7" (provided) is used to jump the vehicle (provided).

Variables

- The height of the ramp is determined by each team.
- The angle of elevation of the ramp is determined by each team.
- The distance between the end of the track and the beginning of the ramp

is determined by each team. See the schematic for further details.

Additional Rules

- The vehicle must be released and not pushed at the beginning of the launch.
- The end of the track **must** be in contact with the floor and **must not** make contact with the ramp.
- Judges will determine the horizontal distance traveled from the terminus of the ramp to where the vehicle first makes contact with the floor.

An illustration of the Jump Schematic is included at the end of the article.

A second, and more valuable, prize is typically awarded to the team that explores the relationship between the three variables. For all analysis, the distance of the jump remains as the dependent variable. Presentation requirements and evaluations include the following:

Presentation Requirements

- For each of the three variables, teams collect data and represent it in tabular and graphical form.
- With the aid of a graphing calculator, teams must create three separate equations (one for each variable) that predict the distance of the jump as a function of one of the variables (ramp height, ramp angle, and distance between track and ramp).

Presentation Evaluation

- Teams are given approximately 10 minutes to present their findings prior to executing their single jump.
- Team presentations are scored based upon three criteria:
    1. Data organization/presentation (40%)
    2. Analysis of data (via best-fitting functions) to determine optimum jump configuration (40%)
    3. Clarity and the convincing nature of team presentations (20%).

Some Instructional Tips and Strategies

From a mathematical perspective, this project pushes students to develop the fundamental principles of elementary mathematical modeling. Some teachers have created laboratory worksheets where teams carefully record the data for each variable (jump distance versus ramp height, jump distance versus ramp angle, and jump distance versus ramp distance from track). Graphing calculator technology now makes it easier for students to participate in the process of modeling, assuming roles very similar to those of scientists and engineers. The process of experimentation, data collection, curve fitting, and prediction is repeated at least three times in this project and provides students with a meaningful applications of standard topics in trigonometry and pre-calculus.

In general, feel free to structure the project as you see fit. Some teachers have allowed students to 'explore' with few requirements except that the winning presentation includes the most convincing argument for their final track/ramp configuration. Others require answers to specific questions generated in the teacher-specific laboratory and handouts. In most cases, however, it is best to at least expose students to the data input and regression capabilities of the graphing calculator. Often, I include a brief handout that describes the major keystrokes involved in inputting data, plotting data, and creating lines of best fit. A few fun (and usually manufactured) data sets accompany the handout (amount of sleep versus test score, age of truck versus value, etc.) to provide an opportunity for practice.

The adaptability of the project is also a benefit. If time constraints are a concern, shorten the assignment to a single variable – although I enjoy including the angle of elevation and the height of the ramp as two possible variables because presumably the curves will share much in common. The one-shot, Knieval-esque jump is always fun and takes very little time. You can always orchestrate the amount of time spent on research and testing. Data can be collected a few times at the end of class over a few weeks or it can all be collected during a single period or two. Many teachers prefer the former so that they can 'hype' the event and allow teams to discuss strategies and methodology in the days leading up to the jump. Regardless of your approach, the project ensures that students will apply modeling principles and present their findings to peers in a structured format.

References

Bowman, J. K., Koirala, H. P., and Edmond, L. E., 2000. Graphing Calculators: Critical Tools for Actively Teaching Math and Science. *Computers in the Schools* 16: 135-149.

Kahanec, F., 1985. Mathematics Contests. *Mathematics Teacher* 78(1): 64-65.

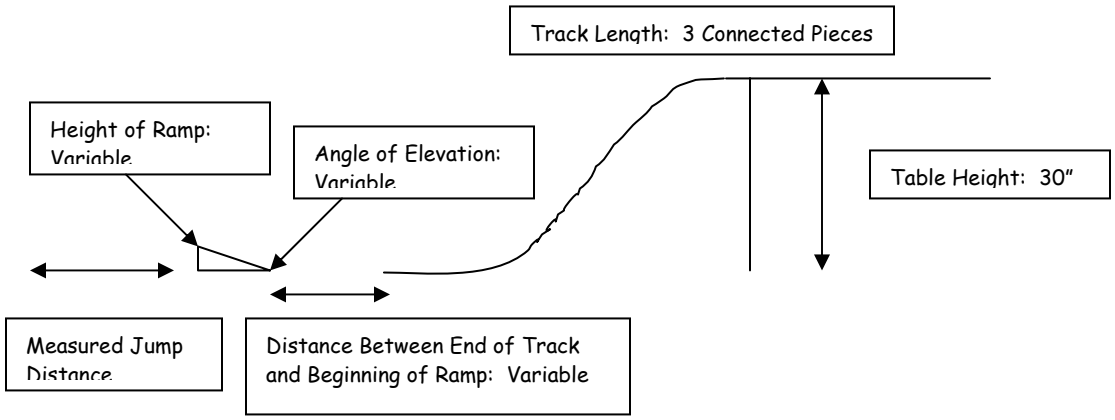
Koes, M. T. and Saab, J. F., 1996. Where All Are Winners: A Mathematics Olympics for Parents, Students, and Teachers. *Teaching Children Mathematics* 3(3): 118-121.

Lapp, D. A., 1999. Multiple Representations for Pattern Exploration with the Graphing Calculator and Manipulatives. Mathematics Teacher 92(2): 109-113.

Mingus, T. and Grassl, R., 1997. Using technology to Enhance Problem Solving and Critical Thinking Skills. *Mathematics and Computer Education* 31(3): 293-300.

National Science Teachers Association. 1990. *Science and Math Events: Connecting and Competing*. Washington DC: NSTA Publications.

Jump Schematic



Joey Chitwood's Thrill Show Revisited and Applications of Mathematical Modeling
Laboratory/Activity Analysis

School: _____

Names

1. _____ **2.** _____

3. _____ **4.** _____

Variable #1: Ramp Height

Hypothesis
(Speculate on what your team expects to be the optimum ramp height to produce the greatest leap. Why?)

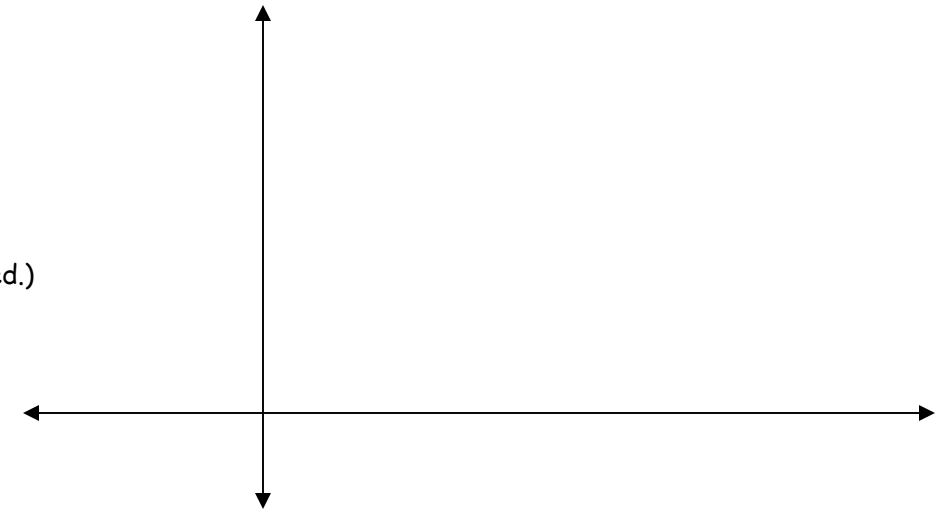
Data Collection
(Complete the table below to document the data that your team has gathered.)

Ramp Height (x): **0**

Jump Distance (y):

Data Analysis

Graph
(Plot your data
on the axes provided.)



Data Analysis Continued

Function
(Use the regression feature on your calculator to create an equation that can be used to model your data. Why did you choose this type of function? According to your function, what is the optimum height of your ramp. How did you determine this mathematically?

Variable #2: Ramp Angle of Elevation
(Explain why you should not have to repeat the experiment for this variable, given your results from Variable #1.)

Variable #3: Distance Between Ramp and Track
Hypothesis
(Speculate on what your team expects to be the optimum distance between the ramp and the end of the track to produce the greatest leap. Why?)

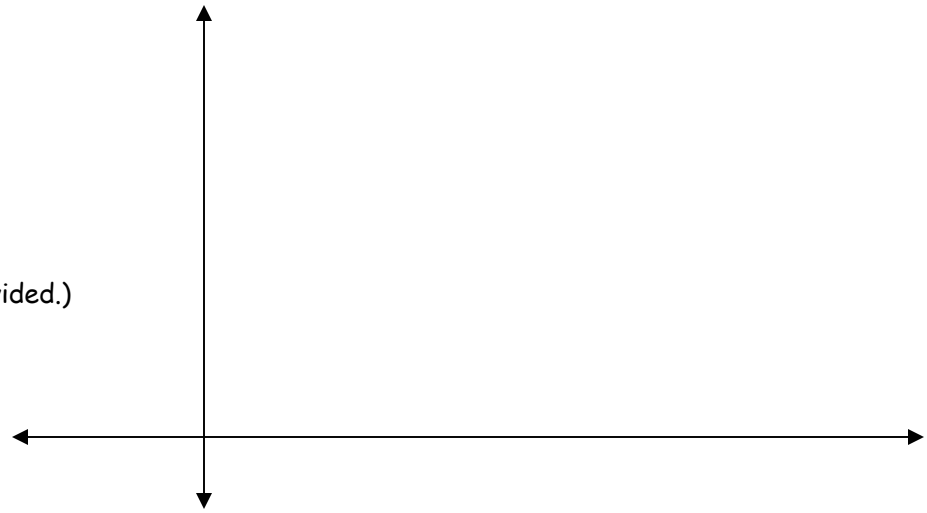
Data Collection
(Complete the table below to document the data that your team has gathered.)

Distance Between
Track and Ramp (x): 0

Jump Distance (y):

Data Analysis

Graph
(Plot your data
on the axes provided.)



Data Analysis Continued

Function
(Use the regression feature on your calculator to create an equation that can be used to model your data. Why did you choose this type of function? According to your function, what is the optimum distance between the end of the track and the location of the ramp. How did you determine this mathematically?

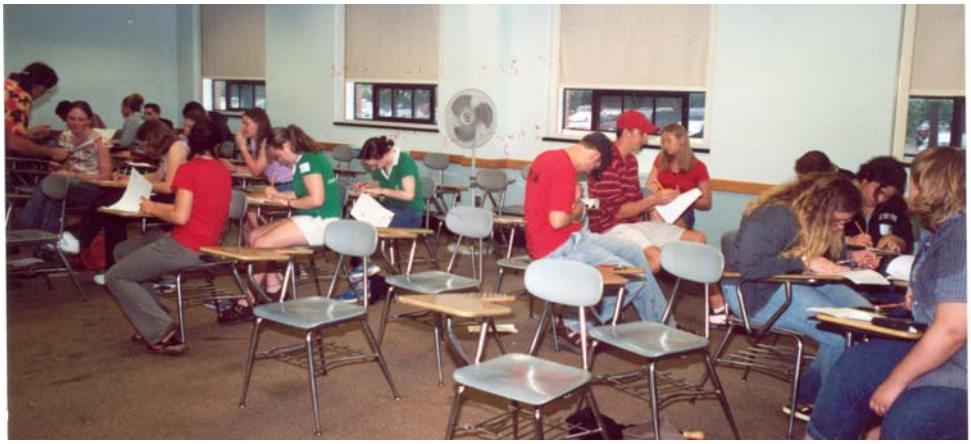
Your Solution
(Precisely where will you set the ramp? Include ramp height, ramp angle of elevation, and distance between the end of the track and the ramp.)

Pictures from Joey Chitwood's Thrill Show Revisited

Final preparation for the One Shot/One Jump Winner Take All Joey Chitwood Math Modeling Challenge

Last minute modifications and set up for the final jump

Measuring the length of the jump



Students working on the lab write up and preparing calculations for their final jump

Learning from Japanese Lesson Study in Southern Illinois[1]

Ann R. Taylor and Laurel D. Puchner
Southern Illinois University Edwardsville
Edwardsville, Illinois 62026

From February 2002 through November 2002, Laurie and I worked with a group of 17 elementary teachers and four math advisors using a version of Japanese Lesson Study for professional development in four school districts in Southern Illinois. Southern Illinois University Edwardsville (SIUE) was part of a consortium of Illinois universities who received a grant to explore lesson study from the Illinois State Board of Education. We have previously reported (Taylor & Puchner, 2002) that participating teachers had found the experience motivating and beneficial. In this paper, we want to share some additional features of the groups' experiences with lesson study. Following a description of Lesson Study, we will organize comments about key happenings for our Illinois lesson study groups under the main steps of the lesson study process, which allows us to simultaneously describe the process and report on the groups' responses to the process.

What Is Lesson Study in Japan?

Lesson study is the centerpiece of Japanese elementary teachers' professional development (Lewis, 2000). Teachers take part in lesson studies within a school, across a district, or in large public demonstration lessons in Japan. Three of our Illinois groups were within schools and one was across two districts. A lesson study involves a group of 3-5 teachers collaboratively researching and planning a lesson, teaching the research lesson, and then discussing the results in a debriefing session (Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). The purpose is to study and think about all aspects of teaching and learning. Many useful resources now exist for educators wishing to begin their own lesson study groups (Fernandez & Chokshi, 2002; Lewis, 2002).

Setting Up Lesson Study Groups

Seventeen teachers took part in the project (four groups each comprising four to five teachers); these teachers taught in traditional urban, suburban and rural settings. After making an initial contact via e-mail, and providing a packet of directions and resources to each group member, we (project facilitators Ann and Laurie) attended initial teacher meetings at school sites in March 2002 to help clarify the lesson study process. We also provided a math advisor (county/district math coordinator or SIUE mathematics department faculty member) to make occasional visits to the meetings to provide mathematical support, and then to attend the teaching of the research lesson and the debriefing.

Each of the four lesson study groups had different levels of knowledge and experience about lesson study, as some group members had read about and completed a previous lesson study, while others knew nothing about the process. Table 1 summarizes the teachers' grade

[1] This work was made possible through a Scientific Literacy Grant from the Illinois State Board of Education. SIUE was part of a consortium of Illinois universities coordinated by Illinois State University.

levels, prior lesson study experience, and lesson topics.

The groups also had different motivations at the outset. The relatively experienced Labelville group recalled their first experience of lesson study two years earlier during a graduate math methods class with Ann: "We felt intimidated, defensive, frustrated." In the class they had studied Japanese lesson study and also read how Chinese teachers had a more profound understanding of elementary mathematics than US teachers (Ma, 1999). When Ann assigned them a lesson study project for the course back in 2000, their primary goal was to prove this research wrong. By March 2002, as grant participants, they were humble and enthusiastic converts to the process (Taylor, Anderson, Meyer, Wagner, & West, 2002; Taylor & Puchner, 2002), happy to receive the support of a small stipend ($500), and the benefit of substitute teachers for the day of the research lesson teaching and debriefing. By contrast, the Ellstown group knew nothing about lesson study. Gillian of Ellstown describes her initial response as "skeptical" of the value of spending so much time planning just one lesson. Nonetheless, with the enthusiastic lead of a county math advisor they all knew, the four Ellstown teachers followed the planning process.

Choosing a Topic

In Japan a topic for the lesson study is chosen by the lesson study group, but it links to an overarching national, district, or school goal. (For example, teachers may work on a lesson study topic of subtraction with regrouping as part of a larger goal of improving children's independent problem-solving.) We found that selecting this overarching goal was very hard for the groups, as they were not used to going through the process of researching together as a district or school to identify goals, nor to using large goals to guide lesson planning decisions.

The experience of the Gresham group, who comprised members of a graduate class taught by Ann, illustrates the potential usefulness of goal development. Ann's facilitation (as course instructor) is one of the reasons they developed a goal while some of the other groups did not. Even though the development of their goal was challenging, (even with Ann's help), it proved to be a useful reference point later on as the group began to worry about their choice of topic and the capability of their students. Half way through their lesson planning they wondered whether they should have chosen to do fractions with first graders, as they weren't sure the children were capable. Referring back to their overarching goal of "Developing adaptive reasoners with the capacity for logical thought, reflection, explanation, and justification" helped them to recognize that it didn't matter that much whether their students got all the right answers, as long as thinking and reasoning occurred. As they wrote in their planning meeting notes: "We need to keep in mind our intentions. Our overarching goal is to develop adaptive reasoners." They also shifted their own emphasis from getting right answers from students to becoming inquirers themselves.

Planning a Research Lesson

The teacher groups spent 10-15 hours planning their lessons, and all of the teachers found this work to be extremely valuable. Although interviews with teachers and observations of meetings indicate that the most important part of the lesson planning was the collaboration and discussion that went on among the teachers, each group did produce a detailed written lesson plan. We had encouraged the teachers to write the lesson using a specific lesson study format (used by Japanese teachers)

that situates the lesson within the larger framework of the unit of study, and describes how the key concepts may be developed in different grades and across a series of lessons. To aid them in the process, we had provided the groups with a multiple column lesson plan format that includes: teacher activities and questions; anticipated student responses in terms of all possible mathematical ideas; and teacher responses to these mathematical ideas. However, only the Ellstown group, strongly supported by the county math advisor, produced this level of completeness in their written plan. A main issue was lack of time, because just talking about one lesson took so long. It is possible that some US teachers will need several rounds of lesson studies before they begin to include such complete attention to breadth and depth in their planning.

Another issue of note pertaining to the planning of the lesson were the very different responses of the groups to the role of an outside math advisor. The Labelville group's advisor only participated in one meeting, which worked fine for that experienced group. Gresham worked together as part of Ann's course, so she served as their advisor/instructor. In two of the schools, however, the outside advisors played a larger role, which interestingly was perceived positively by one group and negatively by the other. In Rose City, the advisor's comments at one of the planning meetings were experienced by the teachers as harsh and critical of their work. These teachers needed to feel in charge of the whole process, setting their own agenda for what and when they wanted to learn. However, the teachers in Ellstown welcomed and encouraged the direct input of their advisor, appreciated her strong lead during their meetings, and responded positively to her critiques of their mathematical and pedagogical ideas. This suggests to us that lesson study advising can be a challenging business in the US. Teachers' needs will vary from group to group, as will their own sense of how much they want advice from others. While we would still see a need for input from mathematical experts, the response of teachers to outside help may need to be carefully monitored, and lesson study facilitators should recognize that what works well for one group may not work for others.

Teaching the Lesson

Japanese teachers regard it as an honor to be the teacher of their group's research lesson. In the US, however, teachers are not as accustomed to making their teaching public. The teachers designated by their peers to teach the research lesson in two of our groups described being very nervous about being observed by colleagues. One of the teachers in particular experienced this as almost debilitating, and refused to allow an additional visitor to observe her. Our research suggests that for some teachers the potential negative effects of being observed, an event that usually occurs in US schools in the context of an evaluation by an authority figure, should be taken seriously by lesson study planners.

While the teacher of the lesson is teaching, the prescribed role of the other group members is to observe and take detailed notes of what the children do and say during the lesson. The teachers in our groups all found it difficult not to behave as teacher-helpers during the research lesson. Prior experience with the Labelville group led us to emphasize the benefits for the teachers of acting as an observer rather than helper, and all of the teachers commented on how much information on student learning they gained from carefully watching and writing down what children did and said during the lesson. We recommend assigning each observer to take detailed notes on a

small group of students in the classroom, rather than suggesting that all observers attempt to watch the whole class.

Debriefing and Learning From What Happened

Immediately following the research lesson in Japan, teachers and observers meet to share feedback and offer detailed critiques of the lesson's strengths and weaknesses. The aim is to understand what happened and to produce an informed revision of the lesson, which may be retaught. However, it should be emphasized that the purpose of lesson study is not to produce a perfect prepackaged lesson to share with others, but rather to learn about and improve teaching as part of a continuous process of professional development.

At the debriefings, all four lesson study groups expressed a deep sense of satisfaction, interest, and delight in the culminating experience of teaching the research lesson. A large part of this satisfaction was related to their observations of the students' responses and mathematical learning. The Ellstown responses are particularly interesting, and reveal the power of lesson study for improving mathematics instruction. The school's usual curriculum is from Saxon mathematics, which emphasizes procedural fluency and provides scripted lessons to prevent teachers making any changes to the set curriculum. The Ellstown group chose to focus their lesson study on teaching through open-ended problem-solving, a method the teacher had tried earlier in the year and "bombed" at completely. Gillian the lead teacher reflects on the group's shock at how well the students responded to the research lesson:

> I was completely surprised, I thought that we had come up with a lesson that was good and …I felt confident that I would be able to teach it the way we had planned... But I was never confident that my kids were going to get this [mathematical thinking]. I still had that huge doubt that my kids just may not understand this type of problem and being a low, low class like they were…[T]hen when it went so well I was just amazed,… I think I said "Who were those kids, they weren't mine!" …They truly amazed me, and it just shows that when you have such good planning and thought process going into your lessons, how much it can affect your students and what you get from them, and even what you think of them… [W]hat I thought they were, was more of a reflection of what I was. And when I turned myself around, it turned them around.

In Gillian's comments, we see three elements of how lesson study may impact teachers. First, Gillian saw a direct correlation between how much time and care the group spent planning lesson activities and the positive response of students, both behaviorally and mathematically. Second, Gillian noticed that students were capable of doing more mathematically than any of their teachers had previously thought. Third, she began to make a much more direct connection between teaching methods and student learning than she had previously, as she recognized that her teaching had been limiting students' learning, and that problems she assumed lay with the students actually lay within herself. These are powerful lessons, and when built on with continued professional attention, they can result in significant changes in teaching methods.

During the lesson study debriefings it was also clear that Ann and Laurie needed to be very careful when offering critique about the lessons. Detailed observations or differences about the effectiveness of a

particular activity are frequently heard as personal criticism. In one specific case, for example, the Rose City group was hoping their first grade students would invent the need to and actually begin to regroup numbers into tens in order to add 18 and 25. However, the teachers chose to use individual unifix cubes as a manipulative, thereby inadvertently removing any need for children to regroup because they could count on by ones or other numbers. The teachers did not notice this conceptual problem with their lesson, and we felt that it was inappropriate to make this a key point of the debriefing. This suggests to us that a) in the case of some teachers, until they are more familiar with and welcome a culture of critique, some mathematical and pedagogical issues may be best not raised by outsiders, and b) some teachers may be more open to hearing some forms of critique in written form after the debriefing. For example, carefully constructed written comments offered in table form after the process was over, and encouraging teacher responses to each of our interpretations, appear to have been effective for the groups in this project.

Conclusion

Our experience with lesson study indicates that this model of professional development can be an effective way for teachers to engage in a study of teaching and learning. We believe that our experience is especially promising since at the university level we took a relatively hands-off approach, such that teachers and/or other school district personnel took the lead. On the other hand, we also discovered that special attention may need to be given to certain aspects of the process that may be particularly difficult for some teachers, such as exposing one's teaching to others, and keeping larger curriculum goals in mind when planning lessons.

References

Fernandez, C., & Chokshi, S. (2002). A practical guide to translating lesson study for a U.S. setting. *Phi Delta Kappan, 84*(2), 128-134.

Lewis, C. (2000). *Lesson study: The core of Japanese professional development* (Invited Address to the Special Interest Group on Research in Mathematics Education). New Orleans: American Educational Research Association Annual Meeting.

Lewis, C., & Tsuchida, I. (1998). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator*(Winter), 14-17,50-52.

Lewis, C. C. (2002). *Lesson Study: A Handbook of Teacher-led Instructional Change*. Philadelphia, PA: Research for Better Schools, Inc.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.

Taylor, A., Anderson, S., Meyer, K., Wagner, M. K., & West, C. (2002). Lesson study: A professional development model for mathematics reform. *Rural Educator*, Submitted May 2002 Under Review.

Taylor, A., & Puchner, L. (2002). Using lesson study for professional development. *Illinois Mathematics Teacher, 53*(1), 23-28.

Table 1: SIUE Lesson Study Groups: Preliminary Summary

Group Grade Levels	Advisor	Lesson Grade	Lesson Goal	Research Lesson Topic & Lesson Problem
Gresham Grade 6 Grade 3-4 Title Grade 1 Band P-3 pre-service	SIUE Ed Faculty	1st	"Adaptive reasoning – capacity for logical thought, reflection, explanation, and justification" (Adding it up, p. 116)	Ordering fractions by size: ¼ ½ 1 *Miss Betty has a problem and she needs our help to solve her problem. She is making cookies; she needs ½ cup of sugar, 1 cup of flour, and ¼ cup of chocolate chips. Miss Betty will use the ingredients in order from the smallest to the largest amount. What should she use first, next, and last?*
Ellstown Grade 4 Grade 4 Grade 3 Grade 3	County Math and Science Coordinator SIUE Ed Faculty (Ann)	4th	Individuals use critical and higher level thinking skills to solve open-ended problems	Open-ended problem solving *Farmer Brown saw 40 heads in the barnyard, some were chickens and some were pigs. He counted 100 feet. How many of each animal did Farmer Brown see?*
Rose City Grade K Grade 1 Grade 1 Grade 2	County Math Coordinator SIUE Ed Faculty (Ann)	1st	Active engagement of students	Addition with regrouping *Mrs. Bateman and Mrs. Morgan need to buy soda for the A.R. party. Mrs. Bateman has 24 students. Mrs. Morgan has 18 students. How many sodas do we need?*
Labelville Grade 2 Grade 2 Grade 3 Grade 3	SIUE Math Ed Faculty SIUE Ed Faculty (Laurie)	2nd	Teaching two step word problem	Division *A South School 2nd grade class has been studying farm life. They will be taking a field trip to a farm.* *The class will be divided into 6 groups. Each group has 5 girls and 3 boys.* *How many more girls than boys are going?*

Teaching with Cross Sums

Raymond Robb
McKendree College
Lebanon, IL 62254

Introduction

Cross sums are the mathematical equivalents of crossword puzzles. A basic crossword puzzle shape is given, but the horizontal and vertical clues are numbers, not words. The purpose is to fill in the squares with numbers, subject to the following rules:

1. You are only allowed integers between 1 and 9, inclusive.
2. The digits must add up to the sum given.
3. No digit may repeat in any row or column.

Cross sums are readily available from numerous sources. One option is to order books of them from Dell Magazines or Penny Press. (Books of 100 can be purchased for about $5.00 by mail.) Another option is to get them online. A cross sum generator is for sale, though the author has never purchased it, at http://home.attbi.com/~twestbom. There are also free puzzles available on the World Wide Web for printing or solving online. Printable sites include The Puzzle Corner, at http://leopard.kitfox.com/puzzlecorner/puzzles/, and Number Puzzles, at http://www.numberpuzzles.com/freexsum.cfm. Number Puzzles also has a (much shorter) summary of many of the techniques listed below. Sites with Java applets include Puzzle Palace, at http://www.puzzle.gr.jp/puzzle/kakro/applet/index.html.en, and Puzzle Japan, at http://www.puzzle.jp/index.html.en (look for "Kakro").

An Introduction to Proof

A common complaint among students being exposed to proofs for the first time (say, in a high school geometry class) is that proofs are "a waste of time". After all, if something looks true, it must be true.

Anyone taking this belief into solving cross sums doesn't keep it for long. The reader is invited to look at any of the hard puzzles on the pages above, or even the simple ones below. Just fill in some numbers that happen to add to the given clues. You'll get stuck very quickly.

The only way to advance toward solving a cross sum is to not fill in any blank until one is completely, utterly sure of the correctness of one's entry. In other words, it is necessary to prove that an answer is correct before writing it down. A student can reasonably be required to construct a convincing argument that each number filled in is correct, based on the clues and the student's previous answers.

The sections below detail several proof techniques that are useful to veteran cross sum aficionados. Several derive from a general "number sense"; others derive from slightly more advanced mathematics.

Basic Logic

People new to cross sums quickly learn to recognize easy sections – places where the clues offer a small set of possible answers. Figure 1 shows a typical good starting place.

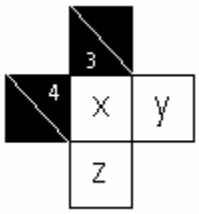


Figure 1: Basic Logic

(Usually the squares aren't labeled with letters; they are now to make the presentation clear.)

The question is: what is x?

Looking across, we know that x+y=4. Since x=y=2 is illegal (rule 3), we must have either 1+3 or 3+1. So x=1 or x=3.

Looking down, we know that x+z=3. Given our bounds we must have 1+2 or 2+1. So x=1 or x=2.

To recap: we know "Either x=1 or x=3," and we know "Either x=1 or x=2." Now clearly x=1, y=3, and z=2.

Proof by Contradiction

When first learning proof by contradiction, students are often quite uncomfortable. "Isn't it pointless to start with the opposite of what we're trying to prove?" is a common refrain. What these students fail to recognize is that a contradiction proof is really a way of knocking out possibilities. Perhaps a better way of introducing them is to call them "what if" proofs.

Consider Figure 2.

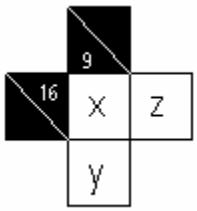


Figure 2: A Contradiction Proof

Looking across, we realize that either x=7 or x=9. Not enough information yet.

So what happens if x=9? Well, z=7. Nothing wrong there. But y=0, which is impossible (Rule 1). Therefore x cannot be 9. We conclude that x=7, y=2, and z=9.

Excluded Digits

Figure 3 shows a different kind of contradiction proof.

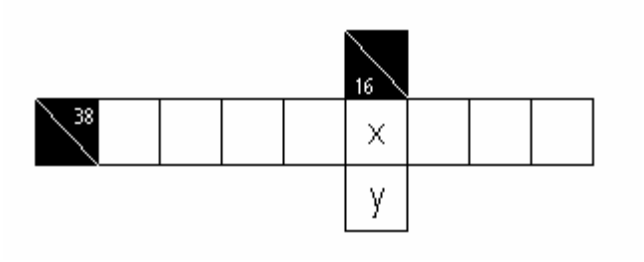


Figure 3: Excluded Digits

How does one get eight digits to add to 38? First, we recognize that 1+2+3+4+5+6+7+8+9=45, a nine digit sum. The only way to get eight digits to add to 38 is to leave out 7. Therefore x cannot be equal to 7.

Looking down, we see as before that x=7 or x=9. We can therefore conclude immediately that x=9 and y=7.

There currently isn't enough information to fill in the other entries in the top row (other than knowing that none are 7 or 9). Frequently, in solving a cross sum, one must leave a section temporarily until more digits are known in other areas.

Parity Check

Figure 4 contains two examples that illustrate the use of parity arguments.

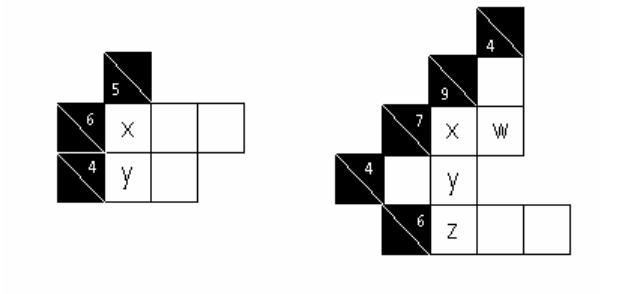


Figure 4: Two Parity Checks

In the example on the left, we see in the first row three digits adding up to 6. This means that they must be {1,2,3} in some order. Looking at the second row, we see that y is either 1 or 3. Therefore, x=5-y must be even. Since only one of the three options is an even number, we have proven that x=2.

The second example is somewhat more complicated. Since w is either 1 or 3 (odd), we know that x must be even. We also know that y must be odd. Therefore, since x+y+z is odd, z must be even. Only z=2 gives the column a proper parity.

Using Inequalities

Figure 5 shows one way to use inequalities to attack a cross sum.

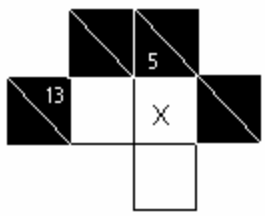


Figure 5: Simple Inequalities

Looking across the row, x is greater than or equal to 4. Looking down the column, x is less than or equal to 4. So x=4.

Cross sums can also be used for practice with reversing inequalities. Figure 6 is an example.

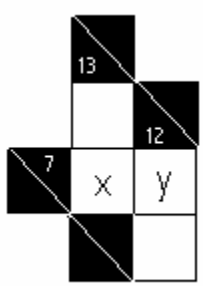


Figure 6: Reversing Inequalities

The first column reveals that x is at least 4. Since x+y=7, we have that y is at most three. However, the last column shows that y is at least three. So y=3, and the rest is easy.

Linear Equations

This example is a bit more complicated, but quite powerful.

Sometimes when solving a cross sum we see an isolated region – one with only one outlet to the rest of the puzzle. Such a formulation often gives us the opportunity to solve for the one square that sits on the border between the region and the rest of the puzzle, splitting the puzzle into two sub-puzzles. Figure 7 is an example.

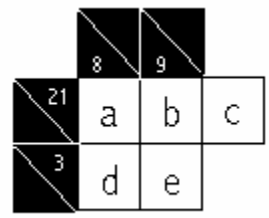


Figure 7: Linear Equations

Looking across the rows in this example, we see:

a+b+c=21

d+e=3.

Looking down the first two columns, we see:

a+d=8

b+e=9.

Combining the first two yields a+b+c+d+e=24. Combining the latter two yields a+b+d+e=17. Therefore we conclude that c=7.

In fact, the shown portion can be completed, since d=1 leads to a=b=7 (breaking Rule 3). We therefore get Figure 8:

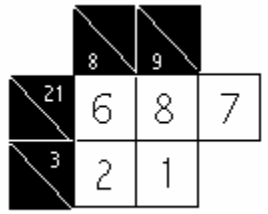


Figure 8: Previous Example, Solved

This technique can be expanded to encompass more complicated structures. For example, consider Figure 9 below.

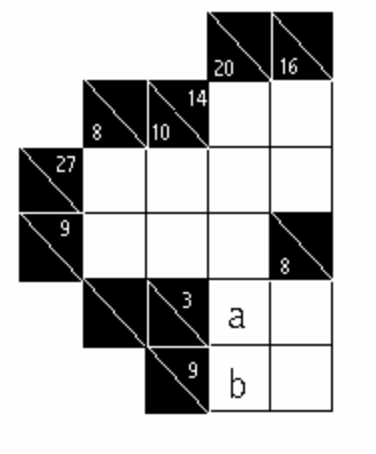


Figure 9: Advanced Linear Equations

First, determine what a+b is. Then determine what a and b are. A solution to the whole figure is available at the end.

Another application of this method is in Figure 10.

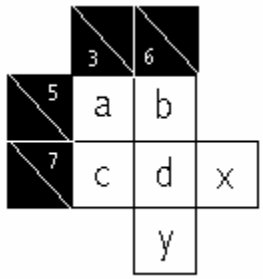


Figure 10: More Advanced Linear Equations

The two rows produce a+b+c+d+x=12. The two columns produce a+b+c+d+y=9. Subtracting the second from the first produces x-y=3. Since x is either 1, 2, or 4 and y must be positive, we can solve for x and y, and then finish the puzzle.

The Perils of Making Your Own

It is surprisingly difficult to construct your own cross sums, which makes the presence of free puzzles online (and cheap ones by mail) quite fortunate. Two main issues arise.

First, when can one be sure that a solution exists? Sometimes one doesn't, as in Figure 11 below:

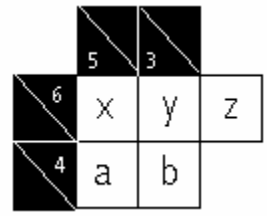


Figure 11: No Solution

Here a is 1 or 3. Since x is at most 3, we know a is at least 2, so a=3 and b=1. But that forces x=y=2.

Second, is the solution unique? Sometimes it isn't, as in Figure 12 below:

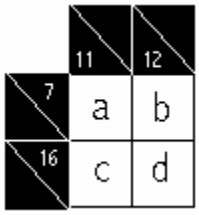


Figure 12: Multiple solutions

Two sets of possible solutions are: a=2, b=5, c=9, d=7, AND a=4, b=3, c=7, d=9.

It is important to note that both problems are fatal to a good cross sum. A multiple solution case makes it impossible to "prove" that your answer is right, because another possibility always exists.

It is possible to avoid the first problem by carefully generating the intended solution and your clues simultaneously. The second issue, however, is difficult to overcome. This author needed several attempts before successfully constructing a cross sum the size of figure 9.

Solution to Figure 9

The thought plan below is how I solved the larger cross sum. Your reasoning may have been different. Students will frequently use original methods to produce correct solutions.

First use the techniques of section 8 to prove that a=1 and b=3, and then finish the lower square. Then solve the upper right squares. (16 = ?) Next, figure out what the two remaining squares must be so that the long column adds to 20. Finally, crack the last four squares. You will ultimately get the finished cross sum shown to the right.

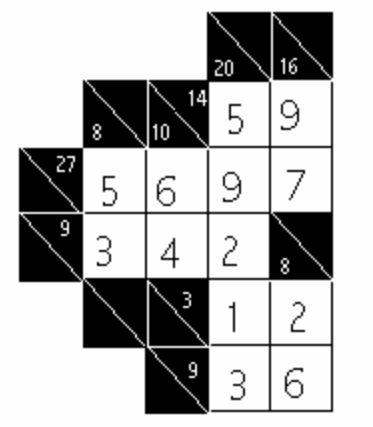


Figure 13: Figure 9, solved

Teaching Math through Just Doing It

Percentages Through Pictures

Shyla McGill, Ann Hanson and Peter Insley

"Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge." This is the Learning Principle from the NCTM Principles and Standards. This article will provide one example of applying this principle in teaching percentages.
(Principles and Standards, National Council of Teachers of Mathematics, 2000, page 11)

Percentages are often a difficult concept for students to absorb mentally. Instead of drilling students on how to convert decimals to percentages and identifying equalities between fractions and percentages, we recommend giving students an opportunity to create a percentage. We ask students to outline a block of squares on a sheet of graph paper. Then we ask the students to shade in a portion of that block. The lesson doesn't begin with the word 'percentage' or any numbers or formulae; the lesson begins with a picture that the class creates. It is only after the class has created a picture of a percentage that we ask them to find the numerical value of that percentage. "What percentage of the block did you shade?" When the students see a percentage of their own making, the problem becomes personal, and suddenly the term percentage has an association to something they understand: the shaded portion of graph paper. Thus we begin "actively building new knowledge from experience and prior knowledge."

The supplies for this activity are an overhead projector, overhead markers, graph paper, and the same graph paper copied onto

several transparencies. Begin by asking for a volunteer to come to the overhead and outline some grouping of boxes on one of the transparencies. Ask the class to duplicate the transparency outline on their graph paper at their seats. Ask for a second volunteer to shade in some block of squares within the outline on the transparency on the overhead. Then ask the question: "What percentage is shaded?" The first time we did this, the student outlined a box that was 13 by 20. The second student shaded a block inside that box which was 7 by 7. The unanimous answer of the class was 49 %. The class was certain of their accuracy.

We wanted the class to realize their mistake, so we set that transparency aside and asked for another volunteer. This volunteer drew a two by two block and shaded one square. The first response to the percentage question was 1%. We stood still and waited to see if everyone agreed. One student questioned the problem: "Wait, are we suppose to say how much of the block is shaded – because didn't he shade one fourth?" We raised our eyebrows, scanned the room, shrugged our shoulders and waited again. A few more in the class agreed that one fourth was shaded, which would be 25%, not 1%. Then we put the first drawing back on the overhead. We asked if this was still 49%. The answer was still unanimous.

We went to a third transparency, the student drew a one-by-ten box and shaded in seven squares. "What percentage is shaded?" The response was split between 7% and 70%. We stepped back, letting the class dispute the problem. Several students presented excellent explanations why 70% was correct and why counting the shaded squares couldn't be the percentage in this case. Prior knowledge and the experience with the two-by-two block was building new knowledge. Soon the entire class had expanded their understanding to include the 70% answer. Again we put the first transparency back on the overhead. A few of the students who volunteered to explain the 70% were now questioning the 49% answer. Several students were bored and confident that the 49% answer was correct. Suddenly someone shouted out it wasn't 49% that it was 65%. We listened to many crazy answers. The students had realized they had to divide, but weren't certain which numbers to divide. We asked key questions such as "How much would 50% be?" "How close is 49% to 50%?" "If you had to guess, what would be your best estimate of how much those 49 squares represent out of the 260 squares." Finally, the class agreed the shading looked to be somewhere between 20% to 25%. Simultaneously, several in the class determined the way to furnish the exact answer was to divide the number of shaded blocks by the number of blocks within the outline.

The activity took a full hour. It is extremely difficult for teachers to hold back solutions and let students struggle. However, percentages are a big concept, and for students to master the concept, they need first to understand the sense of the concept, and then later, the computational formula comes more naturally. If the concept makes sense, students can go forward in many more directions with greater confidence. In this lesson we looked at fractions, thus causing us to address division and decimals, and reasonability tests for size associated with percentages. We were delighted with the result: "Students [learning] mathematics with understanding, actively building new knowledge from experience and prior knowledge."

Calculating Candy Combinations: A Puzzle Problem

David R. Duncan and Bonnie H. Litwiller
University of Northern Iowa
Cedar Falls, Iowa 50614-0506

Teachers are constantly seeking problems that will attract students' interest and that can be used to exemplify algebraic concepts and skills. Puzzle problems provide a rich source of opportunities in this arena.

The following problem appeared in an airline magazine provided on an airplane flight: If 10 candy canes and six bags of caramels cost \$2.20 and 10 bags of caramels and six candy canes cost \$2.60, how much change will you get back from a \$5.00 bill if you buy 25 candy canes and 10 bags of caramels?

Although trial and error could certainly be used, algebraic processes provide a valuable tool as an aid in solving this problem.

Let *cc*=price of a candy cane in cents.
Let *c*=price of a bag of caramels in cents.

The two conditions can then be written as:

$$\begin{aligned} 10cc + 6c &= 220 \\ 6cc + 10c &= 260 \end{aligned}.$$

Solve simultaneously:

Multiply by 6: $60cc + 36c = 1320$
Multiply by 10: $60cc + 100c = 2600$
Subtract: $-64c = -1280$
Divide: $c = 20$ (20 cents per bag)

Substituting in the first equation:

$$\begin{aligned} 10cc + 6(20) &= 220 \\ 10cc + 120 &= 220 \\ 10cc &= 100 \\ cc &= 10 \text{ (10 cents per bag)} \end{aligned}$$

Checking in both equations:

$$\begin{aligned} 10(10) + 6(20) &= 220 \\ 220 &= 220 \end{aligned}$$

and

$$\begin{aligned} 6(10) + 10(20) &= 260 \\ 260 &= 260 \end{aligned}.$$

Note that the original question has not been answered. The cost of 25 candy canes and 10 bags of caramels would be

$$25(.10) + 10(.20) = 2.50 + 2.00 = \$4.50.$$

Using a \$5.00 bill would result in 50 cents change.

Would your students stop after finding the values of *cc* and *c* or will they read the whole problem and answer it correctly?

Challenges

1. Solve this problem by graphing.
2. Construct and solve other problems involving different prices.
3. Your students need not be limited to only two variables. They should be encouraged to write and solve problems with three or more variables. Matrix methods may be necessary.
4. Can a spreadsheet be used to aid in the analysis and solution of the original problem?

Best Internet Practices for Secondary Mathematics

Melissa Scranton (middle)
Dianna Galante (right)
Patricia Nugent (left)
Illinois State University

One of the greatest opportunities and challenges for educators in recent years has been the introduction of the Internet into the classroom. The Internet has revolutionized the way mathematics can be discussed, taught and explored (Klotz, 1997). However, many teachers who have tried to integrate the Internet into their classrooms have been confronted by the enormity of what's available online, the difficulty sorting through this vast resource, and sometimes by their own limited technical experience and fluency. When appropriate Web sites are identified and incorporated into lessons in a meaningful way, the Internet has the capability to have a positive impact on student learning by promoting higher order thinking skills, encouraging active learning, and exposing learners to real world scenarios. When used to its full potential, the Internet is a powerful resource that can enhance secondary mathematics instruction.

Current Status of Internet Use in the Classroom

The National Center for Education Statistics (Kerry & Isakson, 2001) has surveyed public schools annually since 1994 to determine their level of access to the Internet. The U.S. Department of Education monitors public school progress in providing Internet access by sending out a survey each fall to a new sample of approximately 1,000 public schools. The schools selected were representative of a larger population of schools nationwide. By the fall of 2000 it was found that (a) 98% of public schools were connected to the Internet, as compared to 35% in 1994; (b) 77% of classrooms were connected to the Internet, as compared to 64% in 1999, and 3% in 1994; (c) the ratio of students to computers with Internet access had decreased from 6 to 1 in 1999, as compared to 5 to 1 in 2000; (d) the ratio of students to instructional computers with Internet access improved from 9 to 1 in 1999, to 7 to 1 in 2000; and (e) 54% of public schools with access to the Internet reported that computers were available to students outside of regular school hours.

Since teachers play a primary role in the success of technology implementation, an important question to ask is whether teachers are equipped to use the Internet in the classroom. According to Moe and Blodgett (as cited in National Institute for Education Statistics [NIES], 2001):

1. Almost two-thirds of all teachers feel they are not at all prepared or only somewhat prepared to use technology in the classroom.
2. Almost two-thirds of teachers had never used a computer before being introduced to one in the classroom. These teachers need basic technology training, especially those who are receiving computers and using the Internet in their classrooms for the first time. (p. 39)

Most teachers believe they have some basic computer knowledge. In a recent survey, by the National Education Association (as cited in NIES, 2001), 94%

of members were able to surf the web. However, most educators do not know how to apply their web-surfing knowledge in planning technology-based activities. As a result, professional development is the key to effective classroom Internet use. A look at the computer training teachers did receive showed it was usually too generic to help them implement technology into their classrooms: 96% of teachers reported they only received instruction on basic computer skills. In another national survey, Market Retrieval Data (as cited in NIES, 2001) reported that although 78% of teachers received some technology training in the 1998-1999 school year, this training was basic and lasted only 1 to 5 hours for 39% of the teachers and 6 to 10 hours for 19% of the teachers.

Teachers' Use of the Internet for Classroom Instruction

One research study (Small, Sutton, Eisenberg, Miwa, & Urfels, 1998) used an electronic questionnaire to poll K-12 educators about their use of the Internet for instructional purposes. The categories of educational resources used ranged from broad curricula to single activities. Results indicated that lesson plans were the most sought after instructional resource on the Internet. The data also revealed that most educators utilized several resources (i.e., print, electronic, human) and then adapted what they found to their specific instructional needs. Many teachers reported finding too much information online. As an example, through the Internet service provider Yahoo, a search can generate as many as 30,000 lesson plans. Frequent complaints from teachers when confronted by this "information overload" suggest that it's time consuming, discouraging, and may in turn cause a premature end to an online search.

Now that most schools are connected to the Internet and more and more classrooms are gaining access, efforts need to be directed toward helping teachers make appropriate use of the Internet in mathematics classrooms. To take full advantage of the power of the Internet, we must use it in "non-transpositional" ways (Slavit & Yeidel, 1999). That is, using it for such tasks as simulating data, electronic process writing, parallel problem solving, virtual gatherings, and dynamic display of information (Barker & Hall, 1998).

Development of Internet Lessons

Internet lessons have the potential to improve learning by engaging students in experiences not previously accessible on a large scale. The rapidly increasing access to and use of the Internet in the classroom has created a corresponding need for the development of Internet activities that exploit the potential of technology as a tool for teaching and learning. The challenge is to extend learning opportunities beyond what is possible with traditional instructional materials.

For the mathematics classroom in particular, lesson plans that incorporate the Internet should integrate the principles promoted in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000). First, Web-based lesson plans should place high expectations on all students. Lessons should focus on important mathematics that promotes learning for conceptual understanding. Effective teaching will require pedagogical strategies that use the Internet and other technology appropriately and assess students using a plethora of methods.

When appropriate tasks are selected, the use of the Internet in mathematics courses can support meaningful learning. Its use can clarify key mathematical concepts

by using multiple representations of data – symbolic, graphical, and numerical. The Internet can provide experiences that are not readily available in the regular classroom, thereby engaging the learner, providing authentic tasks, and serving as an unlimited source of information from the real world. Computer-based, interactive multimedia materials that offer learner control and real-time feedback can facilitate active learning of mathematics. The Web learner becomes actively involved in the environment, integrating new experiences, making decisions, selecting strategies, and developing models of what was experienced online.

Irving and George (2000) recommended that an Internet lesson should include the following phases: exploration, analysis, construction, and evaluation. In 1997, Slavit and Yeidel (1999) designed Internet activities to be used in a precalculus course that took advantage of the interactive, dynamic capabilities of the World Wide Web. Their framework follows:

1. Students need to construct understanding of the topic in question.
2. Connections between classroom practice and ideas in Web-based activities are fundamental to the activities' potential impact on students.
3. Students should be required to conjecture, explore, report, and justify. The Web affords an interactive potential that can support these learning behaviors.
4. Web-based activities should be visually appealing, technologically transparent, interactive, contextually based, enjoyable, connected to course content, and conceptual in nature. (p. 30)

Sample Lesson Plans

To illustrate best practices of Internet use, descriptions of six exemplary Web sites with lessons are presented. They represent only a small portion of the excellent sites that are an exceptional resource for the mathematics classroom. In the first lesson students create their own Math Hall of Fame. They are divided into teams of six and each assigned a specific role. Each team is expected to select and justify the choice of six mathematicians who must be representative of mathematicians throughout history. Students can locate biographical information at recommended sites or find their own sites. The activity culminates in a class presentation of their gallery of mathematicians. Lesson plans, suggestions, and much more are provided for the class at the site www.classroom.com/edsoasis.

The Noon Day Project is a historical geometry problem presented using modern-day interactive geometry software available online at http://k12science.ati.stevens-tech.edu/noonday/noon.html. The project describes how Eratosthenes in ancient times measured shadows and used principles of geometry to determine the circumference of the earth. A variety of methods are used in the lesson including computer graphics, interactive applets, paper folding, and videos.

Some of the best practices using the Internet in the mathematics classroom exploit the graphical power of the Internet and require active involvement of the learner. An award-winning site www.seresc.k12.nh.us/www/alvirne.html designed for AP Calculus students and teachers, offers a wide variety of options that can serve any student in or out of class. In addition to guest mathematicians and a problem of the week, students have access to retired test questions and calculator tips. One especially nice feature found at this site

is the student-interactive applets that illustrate some of the most fundamental concepts in calculus.

Two other explorations, available at www.exploremath.com, provide real-time demonstrations of how the functions change through unique multimedia features by allowing the student to adjust constants for each function. One selection focuses on the unit circle and allows the learner to explore the relationship between the unit circle and the trigonometric functions. In a lesson on inequalities suitable for algebra, students can manipulate and visualize the solution sets of linear inequalities through a unique slide bar and shading option.

The fifth site at http://asterix.ednet.lsu.edu/~edtech/webquest/titanic.html brings students face-to-face with the tragic sinking of the Titanic. Through an expansive collection of links, a large database, and suggested resources, students use their statistical knowledge to determine the answer to intriguing questions. For instance, one question asks students to query passenger statistics and to determine if the directive "women and children first" was followed. The site is extensive and provides several lesson suggestions that incorporate video, database queries, and spreadsheet software. Students are encouraged to justify and present their findings to their class.

Evaluation of Classroom Internet Lessons

The Internet contains an abundance of worthwhile information, but there are no consistent and established standards for the quality of material (Caruso, 1998). Anyone can post anything they want, whenever they want. Therefore, educators need to approach information found on the Internet cautiously, always assessing its validity and reliability. Evaluation criteria for websites should include exemplary authority, accuracy, currency, navigation and design, applicability and content, scope, audience level, quality, and awards. Associated with each of these criteria is a list of questions that serve to guide website evaluation.

As more and more classrooms are being equipped with Internet access, what can be done to also increase the number of teachers actually using the Internet? The government report (NIES, 2001) on technology in education stated, "Teachers need more than a quick course in basic computer operations. They need guidance in using the best tools in the best ways to support the best kinds of instruction. And they need something more. They need time" (p. 41).

Teacher Development

Staff development is essential and can benefit two specific types of mathematics educators. A look first at the preservice teachers revealed it was easier to introduce these new educators to Internet use because they felt more prepared when compared to teachers with 20 or more years of experience (NIES, 2001). However, Pelton and Pelton (1998) stated that these novice mathematics teachers had experienced minimal exposure to appropriately modeled computer instruction, which then limited their implementation of technology in the classroom. These same researchers found that staff development in technology and Internet use not only improved preservice mathematics teachers perceived self-efficacy, but also influenced their decisions of when and how to incorporate computer use in their classroom.

Pelton and Pelton (1989) pointed out that many universities now require courses focused on educational computer technology. Many of these programs incorporate technology into their new teacher certification plans. As an example, Brigham Young University initiated a course that addressed the needs of the

preservice teacher. In this study, the goal was to encourage teacher use of the Internet as an instructional tool with the rationale that:

1. It facilitates distribution of information without the use of paper.
2. It gives the students access to the extensive education resources on the Internet.
3. It encourages communication between students through e-mail and participation in the course newsgroup.
4. It provides a model of actually teaching with technology.
5. It helps students to become familiar with the potential of the Internet in education. (p. 82)

As a direct result of this course, students reported an increase in technical knowledge and in their confidence in using the Internet in the classroom. However, there was no evidence from any longitudinal studies to suggest that there was increased classroom use of the Internet or if Internet use was sustained over an extended period.

The second type of mathematics educator, the veteran teacher, felt less prepared than the novice teacher, and this in turn, limited integration of technology and use of the Internet into the classroom. What can seasoned teachers do to increase their use of the Internet? Most courses offered for teachers only addressed basic computer skills such as word processing. These courses did not provide the needed instruction on how to develop necessary instructional strategies that integrate the Internet into the mathematics curriculum. However, some courses available online did address this need. In one such site, www.massnetworks.org, geared for an introduction to the Internet for teachers, the author presented topics such as technical computer jargon, communication, browsing, Internet searching, and classroom integration of the Internet. Although a helpful site, because it was primarily text, it did not expose the user to the interactive capabilities available online.

In contrast, one such course available at the site http://tft.merit.edu offered this dynamic approach to Internet teacher development. Designed with the intent to help teachers who wanted to learn more about integrating the Internet into their current teaching practices, this course allowed teachers to work alone or with other teachers and a facilitator. Modules in the course covered such topics as finding educational materials, selecting Internet activities, and describing new ways of learning online. Another course specifically designed for mathematics teachers used the Internet to increase teachers' knowledge of the NCTM *Standards* (1991). Through the use of a hypermedia version of these professional standards, users could select from menu items that were then linked to a Web page providing an introduction on using authentic classroom activities and materials.

In conclusion, the Internet can provide mathematics educators with a variety of instructional strategies that can enhance classroom experiences. Finding and incorporating these online resources into the mathematics curriculum is one of the many challenges teachers confront when implementing these activities into their classroom. NIES (2001) recognized the importance of teacher development and called it the critical element necessary for the effective use of technology in the classroom. Through the use of timely and innovative teacher development that is focused on teacher's instructional use of technology and the Internet, teachers can become empowered to transform their mathematics classroom into a classroom of the future.

References

Barker, B. O., & Hall, R .F. (1998). *Teaching with the World Wide Web: Internet resources for educators in Illinois Schools* [On-line]. Available: http://www.iira.org/pubs/pdf/new/RRR96.pdf

Caruso, C. (1998). *Facing the challenge of technology integration: A portfolio of processes.* Rapid City, SD: Technology & Innovations in Education. (ERIC Document Reproduction Service No. ED 425 701)

Irving, M., & George, E. J. (2000). From instructive to constructive: A practical guide to implementing mindtools in the classroom. *Proceedings of the Society for Information Technology & Teacher Education International Conference 2000, San Diego*, 14-19. (ERIC Document Reproduction Service No. ED 444 555)

Kerry, R., & Isakson, J. (2000). *The Internet for learning: Moving from promise to practice*. U.S.

Department of Education, Web-based Commission on Education. [On-line]. Available: http//:www.ed.gov/offices/AC/WBEC/FinalReport/WBECReport.pdf

Klotz, G. (1997). *Mathematics and the World Wide Web* [On-line]. Available http//forum.Swarthmore.edu/articles/epadel

National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

National Institute for Education Statistics. (2001). *Internet access in U.S. public schools and classrooms: 1994 – 2000* (NCES Publication No. 2001-071). Washington, DC: U.S.Government Printing Office.

Pelton, L. F., & Pelton, T. W. (1998). Using WWW, usenets, and e-mail to manage a mathematics pre-service technology course. *Computers in the Schools*, 14(3/4), 79-93.

Shotsberger, P. G. (1999). The INSTRUCT Project: Web professional development for mathematics teachers. *Journal of Computers in Mathematics & Science Technology*,18(1), 49-60.

Slavit, D., & Yeidel, J. (1999). Using Web-Based materials in large-scale precalculus instruction. *International Journal of Computers for Mathematical Learning*, 4(1), 27-50.

Small, R. V, Sutton, S., Eisenberg, M., Miwa, M., & Urfels, C. (1998). An investigation of preK-12 educators' information needs and search behaviors on the Internet. *Proceedings of the National Convention of the Association for Educational Communications and Technology, St. Louis*, 401-413. (ERIC Document Reproduction Service No. ED 423 862)

http://asterix.ednet.lsu.edu/~edtech/webquest/titanic.html

http://k12science.ati.stevens-tech.edu/noonday/noon.html

http://massnetworks.org

http://tft.merit.edu

http://www.classroom.com/edsoasis

http://www.exploremath.com

http://www.seresc.k12.nh.us/www/alvirne.html

Do you want to decide what is printed in T*he Illinois Mathematics Teacher*? If so, we would be interested in having you as a reviewer for the articles received by the IMT. Simply complete the form below, tear out this page, fold and secure at the indicated places, stamp, and then return it to us. We would love to include you in the list of reviewers. If you are currently a reviewer, we would like to have you complete the form as well so that all our information is up to date. Thanks, and we hope to be hearing from you soon.

Name _____

Check preferred mailing address, please complete both columns.

____ Home Address	____ Work Address
_____	_____
_____	_____
_____ (City)	_____ (City)
_____ (State) (Zip Code)	_____ (State) (Zip Code)
_____ (Phone Number)	_____ (Phone Number)
_____ (Best time to call)	_____ (Best time to call)
_____ (E-mail address)	_____ (E-mail address)

Please check as many of the following areas as you are interested in reviewing.

_____ EC-3	_____ Calculus
_____ 4-6	_____ Statistics
_____ Jr. High/Middle School	_____ College
_____ Algebra	_____ Technology
_____ Geometry	_____ Teacher Education
_____ Trigonometry	_____ General Interest
_____ Precalculus	_____ Other _____

Tape here to close

---------- fold here ----------

Please put stamp here

_____

_____

_____

Marilyn Hasty and Tammy Voepel
Mathematics Dept. SIUE
Box 1653
Edwardsville, IL 62026-1653

---------- fold here ----------

MEMBERSHIP APPLICATION FORM

____ New Member ____ Reinstatement ____Renewal ____Change of Address

Name: _____

Check preferred mailing address, please complete both columns.

____ Home Address

_____

_____

_____

(City)

_____

(State) (Zip Code)

_____

(Phone Number)

_____

(E-mail address)

____ Work Address

_____

_____

_____

(City)

_____

(State) (Zip Code)

_____

(Phone Number)

_____

(E-mail address)

****

Profession (check one)

____ EC-3 Teacher
____ 4-6 Teacher
____ Jr. High/Middle Teacher
____ Sr. High Teacher
____ Special Education Teacher
____ Community College
____ College/University
____ Administration
____ Student
____ Retired
____ Institutional Member
____ Other _____

Interests (check up to three)

____ Remedial
____ Gifted
____ Teacher Education
____ Assessment
____Certification
____ Multicultural Evaluation
____ Professional Development
____ Scholarship
____ Technology
____ Research
____ Math Contests
____ Other

****

Dues for ICTM Membership

____ Regular member
One Year $ 25
Two Years $ 45
Five Years $100

____ Student Member
One Year $ 15

____ Retired Member
One Year $ 20
Five Years $ 80

____ Institutional Member, one year $70

Dues Money enclosed: $_____
ICTM Scholarship Fund $_____
Math Contest Fund $_____
Total Enclosed: $_____

Send this application and a check or money order payable to the Illinois Council of Teachers of Mathematics

Mail to:
ICTM Membership
P.O. Box 5438
Buffalo Grove, IL 60089-5438

If recruited as a new member by a current member, please list the recruiter's name.